THE MATHEMATICS TEACHER

Volume XLVI JANUARY · 1953 Number 1

CONTENTS

		Page
The Emergence of Mathematical Needs in an Industrial Center	Arvid W. Jacobson	1
The Increasing Importance of Mathematics	Education Committee of the Industrial Mathematics Society	3
Mathematics in the Skilled Trades	Carl J. Carlton	8
Mathematics in Engineering	Robert Schilling	14
Mathematics in a Retail Business	E. C. Stephenson	18
The Need for Mathematics in Business and Industry	A. P. Fugill, W. H. Hulswit, and C. V. Winder	23
DEPARTMENTS		
Aids to Teaching	Henry W. Syer and Donovan A. Johnson	46
Applications	Sheldon S. Myers	38
Book Section	Joseph Stipanowich	58
Devices for a Mathematics Laboratory	Emil J. Berger; N. Anning	50
Mathematical Miscellanea	Phillip S. Jones; W. J. Hazard	34
Mathematical Recreations	Aaron Bakst	55
News Notes		57
Notes on the History of Mathematics	Vera Sanford	41
References for Mathematics Teachers	William L. Schaaf	28
Research in Mathematics Education	John J. Kinsella	52
What is Going on in Your School?	John A. Brown and Houston T. Karnes; S. B. Wenaas, J. R. Boyd	42
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS		
Attendance Record of Summer Meeting with NEA		7
Fourth Delegate Assembly	Mary C. Rogers	32
President's Page	John R. Mayor	31
Registrations for 12th Summer Meeting		49

OFFICIAL JOURNAL OF THE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
1201 Sixteenth Street, N.W., Washington 6, D.C.

Printed at Menasha, Wisconsin, U.S.A.

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930. Printed in U.S.A.

THE MATHEMATICS TEACHER

Official Journal of the National Council
of Teachers of Mathematics

Devoted to the interests of mathematics teachers in Elementary and Secondary Schools, Junior Colleges and Teacher Education

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THE MATHEMATICS TEACHER

Volume XLVI January

Number 1 1953

The Emergence of Mathematical Needs in an Industrial Center*

By ARVID W. JACOBSON

Wayne University, Detroit, Michigan

THE foremost feature in modern science and technology is the expanding role of mathematics. In industry and business the increasing complexity of problems and the ever-present search for better products and services, lead to the use of mathematical methods. Trial and error methods can not alone yield the information of the behavior of a physical system or a business procedure or an economic process necessary if improvement in design or function is to be achieved. To understand and evaluate the effects of small components on the behavior of a system, it must be considered as a single operating unit. The functional dependance of the entire system must be expressed in terms of all of the components, large and small. Thus a mathematical model emerges which is an abstraction of the quantitative and logical relationships of the system. Often, as further improvements are sought, the effect of a larger number of these smaller components need to be understood and weighed. It is thus the proper evaluation of the small effects or "second order effects" that determines progress.

Hence advancement depends on our ability to understand and control systems of increasing complexity. Since mathematics provides the main medium through which the large number of interrelationships can be expressed, it is clear that the mathematical model becomes correspondingly complicated. How then can a solution of this complex model be found? Until recent years such a solution was in most cases impractical. However, a new and powerful tool has now come to our rescue. The appearance of the large-scale, high speed computing machine has made the application of mathematics to involved problems and processes a practical matter. The electronic computer thus multiplies the usefulness and extends the application of mathematics. It in turn increases the need for mathematically trained people. For the first time mathematics and mathematicians have come into their own.

Many individuals in engineering, business and education have felt that steps should be taken to enlarge the role of mathematics in order to meet the needs of more exacting technological standards. Thus some three years ago a new organization, the Industrial Mathematics Society, was formed. The Society engages in a variety of activities in the form of general meetings, orientation lectures and study groups on different phases of industrial mathematics. Its membership of some 250 consists of research engineers, physical

* Presented at the 1952 Summer Meeting of the National Council of Teachers of Mathematics held at Detroit, Michigan, June 30, 1952.

scientists and members of college faculties. The Society has had a significant influence in promoting interest in the application of mathematics in industry.

Among the several committees that carry on the activities of the Society is the Education Committee. During the past year this Committee has been collecting material and studying the problems relating to mathematics education in the schools. This Committee decided to publish a statement based on this study and dealing with the broadening applications of mathematics in industry under the title "The Increasing Importance of Mathematics."

The need for a much larger number of mathematically trained students is a real challenge to us teachers. In our search for ways of raising the general competence in mathematics, we have the task of training an increasing number of students for ever expanding technical fields in which mathematics is a basic tool. The following are a few suggestions which I feel are important in our search for ways to better meet these colossal tasks before us.

I believe that there is room for improvement in the teaching of mathematics. It seems to me that our teaching has been altogether too mechanistic, and almost devoid of intuitive content. The learning process does not involve only the intellect or memory but the intuitive or emotional acceptance of the idea or concept. Perhaps a greater use of teaching aids would be of help here as well as a painstaking development of the concept and the intuitive orientation of the youngster by a constant association of the abstract concept with the physical world and his previous experiences. While many insights come to us as if by revelation, basic concepts need to be cultivated over a period of time for they are the product of growth. Hence deep concepts, such as most arithmetic and mathematical ones, need to be presented repeatedly, each time with a wider association and greater depth, always of course in accordance with the psychological maturity of the child.

In motivation and arousing interest in mathematics, its uses must be constantly and interestingly presented to the students. This requires that teachers know a wide variety of applications and can illustrate the use of mathematics in the solution of practical problems. The use of the laboratory method and the collection of problem material for classroom purposes are worthy of consideration. The work of Professor W. W. Rankin's committee and the Mathematics Institutes he has sponsored at Duke University are significant contributions in this direction. Also the policy of industrial and business fellowships for teachers of mathematics should be encouraged and extended.

In teacher training the following questions call for answers: (a) should teachers of all subjects and at all grade levels be acquainted with basic concepts and principles of mathematics and science? and (b) should teachers of elementary schools be trained in the techniques of teaching arithmetic?

Perhaps the overriding task we are confronted with is to elevate mathematics to the pre-eminent position it should occupy as a dominant factor in our modern life and culture. We need to reiterate the glorious role of mathematics in the development of science, technology and the arts. Fundamentals of mathematics need to become a part of general education just as literature, music or painting.

Today we are faced with a tremendous task of training a vastly larger number of individuals in mathematics than ever before. This we must do if we are going to advance along the road of progress and, more crucially, this we must do if we are to safeguard our national existence. Hence we teachers of mathematics have a grave responsibility. Let us give of ourselves with energy, inspiration and patriotic devotion.

The Increasing Importance of Mathematics

By The Education Committee*
of the Industrial Mathematics Society, Detroit, Michigan

We should like to present to you a point of view which may not have occurred to you before—namely that competence in mathematics widespread among our people is essential for the preservation of our society. Competence in mathematics is necessary not only for the mathematics specialist, but for every one of us who is an active citizen. America and the world are facing enormous problems of progress and survival which are technical in nature and requiring more mathematics from everyone than ever before. It is necessary for some of us to master highly technical mathematics which will be required to solve these problems, and for all of us to know enough mathematics to recognize the problems, to evaluate their importance, and to give support to those who actively endeavor to solve them.

About the year 1800 in the history of our country, there began a long and continuous development and expansion of science and industry. This was paralleled with the construction of the means of trade and communication. A network of canals, railroads, highways, and telegraph lines spread across the country. This activity seemed about to culminate around the year 1900. Many leading thinkers felt that we were approaching a limit of industrial development just as we had reached the nation's physical limits. They thought that we were reaching a plateau on which we would slow down, learn to live more quietly, and solve the problems which had arisen in our industrial society. It seemed that we should learn more about living with our neighbors, fitting democracy to modern needs, and becoming better social citizens. To many people this dictated a lessening of emphasis on mathematics, science, and engineering, and a slowing of the pace of invention, research, and development.

This way of thinking became dominant in our great centers of learning and throughout government circles, and filtered down to the high schools, so that today we find *relatively* fewer students studying mathematics and science than two generations ago.

Now we know that this thinking was just about as wrong as could be. Instead of a lessening of our rate of progress, an enormous acceleration was becoming evident before the first world war and continued after it. During the thirties the foundations were laid for whole new fields of endeavor. Television, better aircraft, plastics, synthetic rubber, and electronics were among the many useful things in development. A business upturn brought numerous items to the production stage just before World War II broke out in Europe. Since the war, if international pressures had not marred the picture, we might well now be in a new golden age of technical applications for the benefit of man.

However, great forces are abroad in the world which seek to destroy our way of life. Our survival poses tremendous problems of defense production and arms research. Both intensive and widely distributed mathematical "know-how" is essential to our safety. Simultaneously, people's wants have increased as everyone has become acquainted with the more

* This report was prepared by the following members of the Education Committee: C. J. Carlton, Ford Motor Company; R. S. Lankton, Mumford High School; and C. J. Pajot, Nash-Kelvinator Corporation. Other members of the Committee were: L. C. Bagby, Lawrence Institute of Technology; Franklin Frey, Cass Technical High School; W. H. Hulswit, U. S. Rubber Company; A. W. Jacobson, Chairman, Wayne University; P. S. Jones, University of Michigan; L. E. Mehlenbacher, University of Detroit; and Robert Schilling, General Motors Corporation.

pleasant life made possible by modern conveniences. This requires more invention, more production, and more mass distribution—which, in turn, require more workers trained to use quantitative data.

The continued movement of people into our cities multiplies our social problems, and here too it becomes increasingly clear that the successful handling of the problems requires mathematical know-how. There is increased need for power lines, water mains, fire protection, improved streets and express-ways, gas service, telephone lines, and schools for more populous and extensive cities. Providing these services continually presents technical challenges, and requires clear thinking and careful measuring of social needs.

Mathematics for the Specialist

At this point we want to present a little of this picture of how the world of industry is shaping up, and to note how vital a knowledge of mathematics is. A few fields now developing and expanding very rapidly will show you the essential role that mathematics will play. It should then be clear that there is a need to have more and more people studying mathematics, and that the need applies to *you*.

Consider the field of machine tool design. It is estimated that in the near future factories will have become nearly automatic. There will be automation in the sense of mechanical handling of parts in production. Already in use are conveyors that carry engine blocks through machines with stops at work stations for operations to be performed without the use of human hands. The increase in complexity and number of machine tools in these factories is almost beyond comprehension. Fewer laborers will be required, but the number of technically trained maintenance workers, engineers, and designers will greatly increase. Clearly all of these must begin early a thorough study of mathematics.

In testing materials, equipment to test whole airplanes or similar large structures is becoming available. An operator can apply loads to any desired point of the structure by pressing buttons. The corresponding stresses and deflections can then be read from a series of dials. This use of strain gauges, photo elasticity, and stress coating represents rapidly increasing technical fields which require people trained in science and mathematics.

The applications of statistics in modern life are expanding very rapidly. This field of work sounds formidable probably because statistics is a hard word to spell and to pronounce. It involves the collection, sorting, arranging, and interpretation of information which can be expressed in numbers. The individual items of information may be useless in themselves; but when a statistical summary is studied and analyzed, it gives an over-all picture of a situation and makes possible intelligent planning and action.

The census of 1950 is probably the biggest statistical job that is familiar to all of us. More types of information were gathered, sorted and analyzed than ever before. The publication of this summary tells us where people are moving. Corporations which want to build new plants or warehouses or to open new stores or distributing points can thus tell where they will be most needed and best able to serve a market. School boards can tell where new schools must be built and how large they must be. The politicians know how representation districts must be changed to give fair representation to those states which have grown most. We could easily list dozens of activities which will be affected by the information gathered and classified in this one survey.

Industry, in the last few years, has very widely adopted the principles of statistical quality control. Very simply explained this means that we measure the normal variations in part sizes coming from a given machine. We can then make a control chart on which sample averages are plotted at regular intervals. When the chart shows the operation "out of control," we immediately try to find the cause and correct it. Space does not permit us to list the many applications of statistics in

industry. But, when we consider that at least fifty times as many statistical quality analysts were employed in 1952 as in 1945, the rapid growth of opportunity in this field is obvious.

Finally we might note that the strategic bombing of enemy targets in the last war was plotted for General Hap Arnold by a battery of statisticians. While our losses in men and planes were bitterly tragic, they would have been much worse, the effects of the bombings would have been less and the war would have lasted longer without the services of this statistical battery.

Electronics is another field which has developed enormously in the past twenty years and which is now expanding even more rapidly. The advances in radio, television and the uses of the "electric eye" or photoelectric cell are popularly described in hundreds of magazine and newspaper articles. Not so well known are large welding machines which have panels full of electronic tubes more complicated than any radio and such things as induction heating furnaces. Here again we do not have space to list the hundreds of uses of electronic tubes.

What is not generally realized is the hugh amount of mathematical work involved in the design, engineering, building, installation, and maintenance of this equipment. Every person who thinks of going into this field of work needs a sound foundation in mathematics. Many men now studying to enter this field find to their regret that they have to slow down their progress in order to study the mathematics which they should have mastered in school.

The advance in many fields of chemistry is creating a need for a vastly expanded number of chemical engineers. Synthetic fibers, silicones, petroleum products, metallurgy, and plastics come readily to mind. Radioactive materials stand at the threshold of wide industrial usefulness. Few people are prepared to work in this field. Fuels for rocket ships, metals that will withstand extremely high temperatures, and problems of heat transfer require considerably advanced mathematics. Synthetic rubber has become an important item in our economy. The extraction of chemicals from the sea and the development of agricultural chemicals, new insecticides and fungicides cannot be accomplished without those who are trained in mathematics.

The man or woman who enters the field of business administration finds that a surprisingly large number of job opportunities require mathematical training. Cost accounting, purchase analysis, price analysis, production programming and control, and other fields in manufacturing require many more people than even a few years ago. In the administration of sales and distribution, we also need a much larger number of people than formerly who have training in fields which are primarily mathematical. To name only a few we have credit policies subject to frequently changing government regulations, price policies similarly regulated, sales and distribution cost control, the expansion of distribution facilities and more complex problems in exporting and importing due to changing money values all over the world. In the strictly financial end of business we have pension and insurance administration, tax accounting, changes in capital structure, and other activities.

Everyone who looks forward to working in the business field should, therefore, build a sure foundation in mathematics in high school to foster his chances for success, advancement, and service later.

The advent of the large-scale computing machine marks the beginning of a new and far-reaching development in science and technology. These machines are at once mathematical computers and information processing media. The application of electronics has increased the speed of computation so that problems of great complexity can be solved in a small fraction of the time formerly required. In fact engineering and research problems are now being done which would have never

been tackled by desk calculators because of the enormous amount of time involved.

Before these high-speed devices can be set to work, the problem has to be expressed in a mathematical form. The main function of the computer is to carry out routine operations and decisions which the mathematician has instructed it to perform. The electronic computers have made the application of mathematics practical and have created a need for a greatly increased number of persons with a mathematical training.

Our present enormous military establishment will continue to require large numbers of technically trained people for years to come. The fact that the armed services absorbed 19,000 of 38,000 engineering graduates in 1951 is potent evidence that this is true. Atomic research, guided missiles, sonar, radar, improved aircraft, improved weapons and a thousand other items in competition with those of the communist world require research, engineering, and mathematics of a most intensive sort. Military personnel who serve in such capacities as navigators, gunners, and in many clerical jobs will do well to bring to their tasks all the mathematical training that they can.

Perhaps the best known area of professional opportunity for which mathematics is the foundation is engineering. Every branch of engineering, civil, mechanical, electrical, chemical, mining, metallurgical and industrial requires the solution of problems by mathematical means.

Henry Taylor, radio commentator, made an exhaustive survey of the engineering situation in 1951. In November of that year he made three broadcasts summarized in a pamphlet "A Great Opportunity Uncovered."[1]

Taylor points out the sad dearth of engineering graduates in 1951 which will grow progressively worse in the next few years. Data on the disparity between graduates and jobs available point most vividly to the need for guiding more young people toward engineering training. Since mathematics is the foundation for engineering, more mathematics training in high school is indicated as an urgent need.

In 1951, 80,000 jobs for new engineers were open; 38,000 engineers were graduated. Nearly 19,000 were drawn into the armed services leaving fewer than 20,000 engineering graduates to fill 80,000 jobs, a shortage of 60,000 engineers. The number of engineering graduates will further decline to about 14,000 in 1954 while the need for engineers will steadily increase.

There are excellent opportunities for engineers in top positions. Forty per cent of industrial management is engineering trained. Ten of the twenty presidents of associated companies of Bell Telephone are graduate engineers. Nearly 50 per cent of the top officials of the General Electric Company started as engineers.

The present shortage of engineers is by no means temporary. *Time* magazine points out that about 50 years ago one engineer was employed in industry for each 250 employees. Twenty-five years ago the ratio had risen to one in 100. Today the trend is continuing. About one in 50 industrial employees is in engineering.

Mathematics for the Non-Specialist

How does all this affect you? Perhaps you are saying you do not intend to be an engineer, a statistical worker, a technician in industry, or a business person who will need to be competent in mathematics. Do not make the mistake, however, of thinking that a mathematical background is only desirable for those who will use it directly. Many doors to the future will be closed to you in hobbies and recreation, as well as in your life's work if you are not well grounded in mathematics. You may develop an interest in photography, but be prevented from following an article even in a popular magazine on this subject because of your mathematical shortcomings. You may cut yourself off from

[1] Copies of the pamphlet may be obtained by writing to the General Motors Corporation, Detroit, Michigan.

future education in many fields in which your interest could later develop. If your chosen occupation does not originally require you to use or understand much mathematics, you may be unable later to utilize or to appreciate fully the new developments that will come in every field. This already has been the experience of many workers in the fields of psychology, medicine, biology, economics, business management, and the social sciences. In your activities as an ordinary citizen you will be more effective in your efforts and less dependent on the judgments of others if you are competent to handle quantitative situations of one kind or another. These will not always be mathematical problems, but you will be using the problem-solving process that your study of mathematics will better prepare you to use. Do not quit the study of mathematics too soon. Interest in it and appreciation of its usefulness will increasingly be yours the longer you persist in its study. You owe it to yourself and to the society in which you live to attain the highest levels of mathematical competence of which you are capable.

Editor's Note

In this issue, we are publishing the papers which were presented at the afternoon session of the 1952 summer meeting of the National Council of Teachers of Mathematics held at Detroit, Michigan on June 30. This session was planned in cooperation with the Industrial Mathematics Society of Detroit and eight members of this Society participated in the program. Since the material presented in these papers emphasized so clearly the need for more training in mathematics at both the secondary and college level, it was felt that members of the Council who were unable to attend this interesting meeting would also benefit from a record of these discussions.

The Industrial Mathematics Society was formed in 1949 and has been especially active in the Detroit area. Its members include research and development engineers, physical scientists who are associated with various manufacturing and service organizations, and university and technical college faculty members. The purpose of the Society is "to bring together persons who use mathematics in engineering in order to learn more about and to extend the field of application of mathematics as well as to develop new procedures for the solution of problems that arise in modern industrial research." Further information about the activities of this organization may be obtained by writing to the Industrial Mathematics Society, 5135 Cass Ave., Detroit 2, Mich.

It is not necessary for us to point out the seriousness of the shortage of persons trained in mathematics and science at the present time. Can this shortage be overcome before it is too late? If so, how?

E.H.C.H.

ATTENDANCE RECORD OF N.C.T.M. SUMMER MEETING WITH N.E.A.

Detroit, Michigan, June 30, 1952

Name of State	No.	Name of State	No.	Name of State	No.
Alabama	5	Kentucky	2	Pennsylvania	5
Alaska	2	Maryland	3	South Carolina	1
California	2	Michigan	49	Texas	2
Delaware	1	Nebraska	1	Virginia	1
District of Columbia	1	New Jersey	1	West Virginia	1
Illinois	3	New York	1	Washington	1
Indiana	3	Ohio	6		
Iowa	1				92

Mathematics in the Skilled Trades*

By Carl J. Carlton

Trade and Technical Training Section, Training Department
Ford Motor Company, Dearborn, Michigan

The shortage of skilled workers in the United States is tending to become a serious bottleneck in the production of consumer's goods and more seriously a handicap in tooling for defense production. A glance through the Help Wanted columns of every newspaper in a manufacturing area makes this apparent to everyone. It might be thought that this is a transitory situation which will ease when the most urgent tooling has been completed for the swelling arms program. A little research into the reasons for the current shortage of skilled men will quickly dispel this belief.

During the thirties when production was at low levels, very few men completed apprenticeships. During World War II the nation's draft policies did not foster the training of skilled workers. As a result we find a bulge of older workers in the age distributions of men in all skilled fields. A study of age distributions in one large corporation disclosed that 34 per cent of all skilled workers will be eligible for retirement in the next ten years. The median age, at time of the study, exceeded 48 years. This situation is typical of all areas similarly studied.

Prior to the first world war and even up to 1924 we drew heavily from the ranks of the skilled workers from Europe who came here in large numbers to benefit from better working conditions and higher wages. This source has diminished to a trickle and offers no solution to our present problem. Another factor tending to aggravate the shortage is the vast expansion of our industrial centers and the creation of new industrial areas across the country. One more factor of a permanent nature is the increasing mechanization of industry which is reversing the trend of some years ago when the needs for both skilled and unskilled workers seemed to be declining and the relative number of semi-skilled workers was increasing. This shortage of skilled men therefore is real, critical and continuing and most unlikely to be remedied for many years to come.

As mathematics teachers we should look at this situation and note what we can do to prepare our students to enter these fields of great opportunity. While in school, students do not understand the place of mathematics in the trades and hence tend to avoid its study because of its rigorous demands and because they see no use for it.

The main purpose of this discussion, then, is to present the applications of mathematics in a few of the many trades in which it is fundamental. Among the common trades are toolmaking, diemaking, patternmaking, industrial hydraulics, industrial electricity, machine repairing, plumbing, pipe fitting, stationary steam engineering, sheet metal working and roll turning and grinding, and that of the industrial machinist and millwright.

Every one of these trades requires plain ordinary arithmetic in its shop applications. Speed and accuracy in calculation and knowledge of a rapid method of checking calculations are all too sadly lacking in many of the candidates for apprenticeship. A mastery of fundamentals is the first step. Operations with common and decimal fractions, square root, the mathematics of common precision instruments, ratio and proportion and percentage are all vital to the shop man. We must also impress the student with the importance of getting the correct solution for a given

* Presented at the 1952 Summer Meeting of the National Council of Teachers of Mathematics, Detroit, Michigan, June 30, 1952.

problem. The superintendent of a dieroom is going to be very unhappy about a thousand dollar die scrapped because of a few incorrect dimensions even though the right method was used in calculation. The method of checking calculations by the excess of nines is very helpful in achieving accuracy.

Algebra seems to have a bad name among students, teachers and parents. The equation, the formula, the use of signed numbers and the advantages of symbols in mathematical analysis are not well understood. The fact that algebra is directly applicable to many shop problems might be a strong motivation for both student and teacher.

An examination of one of the handbooks for any trade will soon make this clear. The problem shown below is typical of those encountered in the dieroom and can be easily solved by algebra. Figure 1 illustrates how a die section is checked. In Figure 2, the measurements to be used in determining the radius R of the plug are shown. In triangle ABO, $AO=2.812$ and $AB=1.000$. Hence

$$OB^2=2.812^2-1.000^2$$

and

$$OB=\sqrt{6.907344}$$

or

$$OB=2.628$$

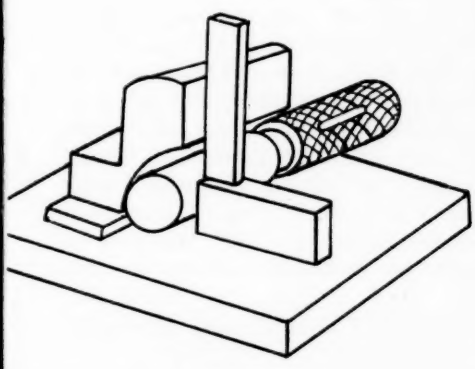

FIG. 1

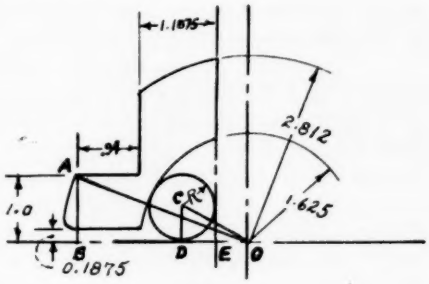

FIG. 2

Now it is possible to solve for R in triangle CDO. Let $OE=y$. Then $OD=y+R$, $CD=R$, and $OC=1.625-R$. Now

$$\begin{aligned} OE &= OB-(.9400+1.1875) \\ &= 2.6282-2.1275 \\ &= .5007. \end{aligned}$$

Using the Pythagorean theorem we have

$$R^2=(1.625-R)^2-(y+r)^2.$$

Hence

$$R^2+4.2514R-2.3899=0$$

and

$$R=.5027.$$

Many applications of geometry may be found in the various trades also. The sheet metal worker and the template maker find geometrical constructions of great accuracy indispensable in laying out their work. Furthermore geometry is the basis for trigonometry and enormously valuable in the mastery of the latter subject. For example the projection of one side of a triangle on another is the basis of the cosine law for the solution of oblique triangles. An interesting application of the Pythagorean theorem is found in Figure 3. In right triangle AOB, $AO=1.38$ and $OB=3.00$. Then

$$AB=\sqrt{1.38^2+3.0^2}=\sqrt{1.9044+9.00}$$

or

$$AB=3.30.$$

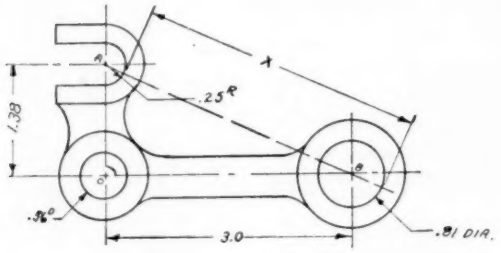

FIG. 3

Hence

$$X = AB - .25 + .405 = 3.46.$$

Trigonometry is the real handmaiden of mathematics for the man in the machine shop and is equally important for the industrial electrician. In the engineering department and the drafting room, production parts or die or fixture details are often drawn with every dimension given to describe their sizes and shapes. But for the shop man to set up the job on the lathe, mill, grinder or engraver, the information is insufficient. This type of work often calls for extreme accuracy. To determine the mechanic's accuracy, the checker on the inspection bench must make additional calculations. This is especially true when the original layout points or check points have been machined away. Since measurements to three ten thousandths of an inch are not at all uncommon and gauges are often made to an accuracy of half a ten thousandth, the importance of accuracy in calculations is obvious.

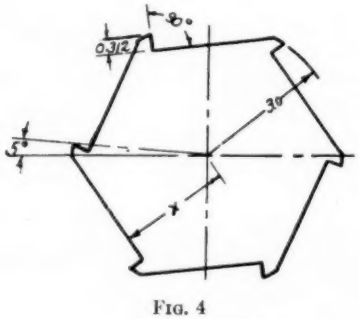

FIG. 4

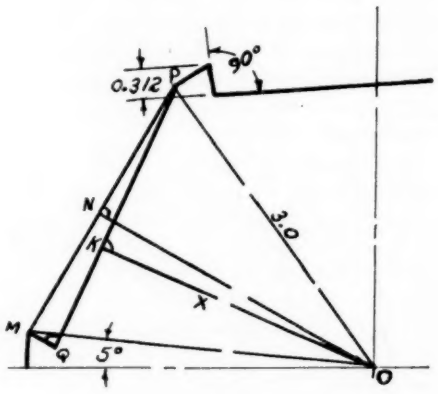

FIG. 5

An examination of Figures 4 and 5 will show the reasons for calculations involving plane trigonometry in the shop even though all necessary dimensions appear on the drawing. In Figure 4, the length of the perpendicular OK which is one half of the distance across flats is to be determined. In triangle OMN, $OM = 3.00$, angle $MON = 27°\ 30'$ and MN can be calculated. In triangle MPQ, MP and MQ are known and the angle MPQ may be found. Finally in triangle KOP, OP and angle OPK are known and hence we can solve for the length of OK. Similarly in Figure 6 we have the problem of locating a grinding center which involves finding the lengths of DO and AD. Applying the cosine law to triangle ABC we have that

$$CB = \sqrt{AC^2 + AB^2 - 2(AB)(AC)\cos 48°}$$

and using the law of sines we find that

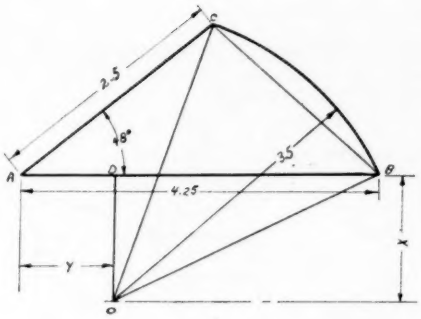

FIG. 6

$$\sin CBA = \frac{2.5 \sin 48°}{CB}.$$

In triangle OCB we have that

$$\cos CBO = \frac{CB}{2(OB)} = \frac{CB}{7.0}.$$

Since $\angle DBO = \angle CBO - \angle CBA$, we can solve for DB, DO and AD.

Shop calculations are frequently necessary to set up jobs in which surfaces or channels oblique to horizontal, vertical or end planes must be machined. In grinding or milling such surface or channel and often in broaching, drilling and boring operations, we find that the angles through which a fixture must be tilted or rotated are none of the angles shown on the drawing of the piece.

The problem resolves itself into a problem in solid trigonometry or, in the language of the shop, a compound angle problem.

The solution of such problems by algebraic and trigonometric analysis is possible for those with mathematical training through solid trigonometry and college algebra. However, by reducing all problems which can occur in the shop into five basic types, we may solve them by "compound angle" solutions which may be handled by students without advanced training.

An illustration of such a solution is found on Plate I while the illustration in Plate II shows how it would be impossible for the designer to calculate these angles unless he knows the exact machine or fixture in advance.

The industrial electrical field presents problems which are no less important. While the electrician usually doesn't work to such close tolerances as the toolmaker, his problems are diverse and require a thorough knowledge of electricity and of mathematics through trigonometry. Problems requiring mathematics above this level are the responsibility of the electrical

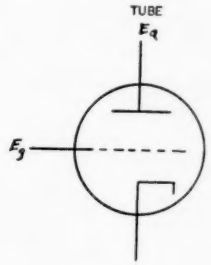

FIG. 7

engineer. One area which has advanced rapidly in recent years is welding. Welding control panels are complex arrangements of electronic tubes which make it possible to apply heat to the parts to be welded continuously or intermittently and in varying amounts as the job requires. The applications are expanding constantly and methods of welding parts of varying sizes and thicknesses are constantly being developed. The electrician must often install controls and solve problems illustrated in Figures 7, 8 and 9.

In the electronic tube, the anode voltage is indicated as E_a and the grid voltage as E_g. If these voltages are exactly in phase,

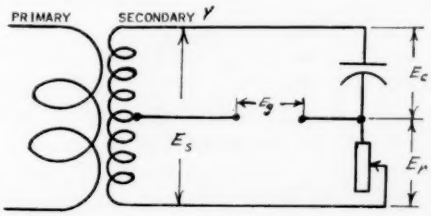

FIG. 8

POSITIVE CLUTCH

THE FOLLOWING IS A COMPOUND ANGLE PROBLEM OF A POSITIVE CLUTCH THE SOLUTION OF WHICH REQUIRES A KNOWLEDGE OF ALGEBRA AND TRIGONOMETRY. THE POSITIVE CLUTCH IS GENERALLY USED IN MANY TYPES OF PUNCH PRESSES.

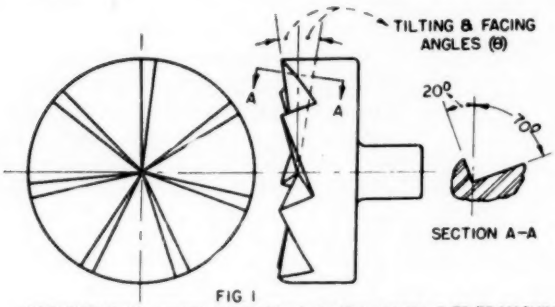

FIG 1
ORTHOGRAPHIC VIEWS OF POSITIVE CLUTCH WHICH IS TO BE MACHINED.

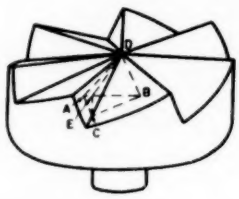

FIG. 2
PICTORIAL VIEW OF CLUTCH.

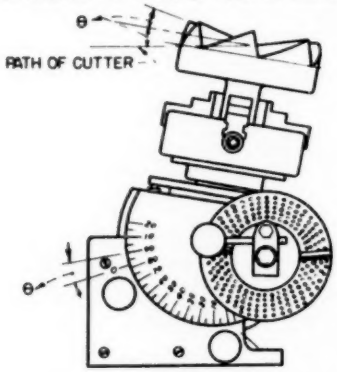

FIG. 3
THE FIGURE ABOVE SHOWS THE CLUTCH MOUNTED ON A TILTED DIVIDING HEAD IN POSITION FOR MACHINING.

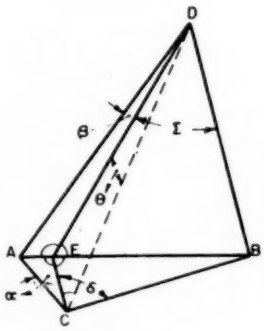

FIG. 4
THE FIGURE ABOVE IS AN ENLARGED VIEW OF THE TETRAHEDRON ABC-D SHOWN IN FIGURE 2.

PROBLEM: DETERMINE THE FACING AND TILTING ANGLE (θ) IN ORDER TO MACHINE SEVEN TEETH IN THE POSITIVE CLUTCH.

N = NUMBER OF TEETH IN POSITIVE CLUTCH $\quad \alpha = 20^0 \quad \delta = 70^0$

-:SOLUTION:-

$\beta + \Sigma = \frac{180^0}{N} = 25^0 42' 51''$ SEE FIGURES 2 AND 4

$\tan(\beta + \Sigma) = \tan 25^0 42' 51'' = \frac{\tan\beta + \tan\Sigma}{1 - \tan\beta \tan\Sigma}$

BUT IN TETRAHEDRON DEC-A, $\tan\beta = \tan\alpha \sin\theta$,
AND IN TETRAHEDRON DEC-B, $\tan\Sigma = \tan\delta \sin\theta$.
BY SUBSTITUTION

$\tan 25^0 42' 51'' = \frac{(\tan\alpha + \tan\delta)\sin\theta}{1 - \tan\alpha \tan\delta \sin\theta}$ SINCE $\alpha + \delta = 90^0$, THEN $\tan\alpha \tan\delta = 1$

$.48157 = \frac{3.11147 \sin\theta}{1 - \sin^2\theta}$

$.48157 - .48157 \sin^2\theta = 3.11147 \sin\theta$

$.48157 \sin^2\theta + 3.11147 \sin\theta - .48157 = 0$

$\sin\theta = \frac{-3.11147 \pm \sqrt{3.11147^2 + 4(.48157)^2}}{.96314} = \frac{-3.11147 \quad 3.25712}{.96314} = .15122$

$\theta = \sin^{-1} .15122 = 8^0 35' 58''$

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PLATE I

APPLICATION OF A COMPOUND ANGLE PROBLEM

DUE TO THE CONSTRUCTION OF THE DIFFERENT MACHINES AND AVAILABLE EQUIPMENT, THE SET-UP ANGLES ARE GENERALLY DIFFERENT FOR THE MACHINING OF A PARTICULAR SURFACE.

THE PURPOSE OF THIS CHART IS TO SHOW THAT THE SET-UP ANGLES DIFFER FOR THE MOUNTING OF THE SAME PART ON DIFFERENT TYPES OF EQUIPMENT, ALSO, DUE TO THE LOCATION OF THE AXES OF THE TILTING DEVICE, THE MACHINING OF RIGHT-HAND AND LEFT-HAND SECTIONS ARE LIKELY TO HAVE DIFFERENT TILTING ANGLES.

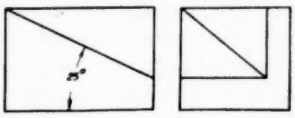

FIG. 1

FIG. 1 SHOWS AN ORTHOGRAPHIC VIEW OF A SECTIONAL BLOCK AND THE MANNER IN WHICH IT IS PRESENTED TO THE LAYMAN.

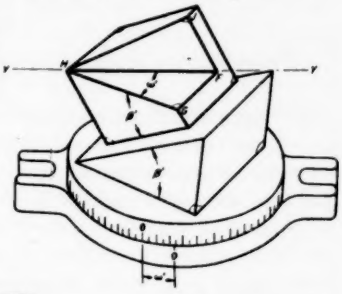

FIG. 2

FIG. 2 SHOWS THE SECTIONAL BLOCK MOUNTED ON A FIXTURE IN PREPARATION FOR GRINDING THE SURFACE HGF.

FIG. 3 SHOWS THE NECESSARY TILTING AND ROTATION ANGLES (β) (ω, I) FOR MACHINING THE SURFACE HGF OF THE SECTIONAL BLOCK ON A SHAPER. THESE TILTING AND ROTATION ANGLES ARE NECESSARY IN ORDER THAT THE LINE YY AND THE SURFACE HGF BE PARALLEL TO THE PATH OF THE TOOL AND THE CROSS FEED OF THE SHAPER RESPECTIVELY.

THE ANGLES OF TILT AND ROTATION (FIG. 3) ARE COMPUTED BY THE FOLLOWING FORMULAS:

ω = 30° (FIG. 1)

TAN β = TAN 25° COS 30° = .4663 ∿ .86603
= .40383
β = TAN⁻¹ .40383 = 21° 59' 24"
TAN I = SIN β TAN 30° = .37445 ∿ .57735
= .21618
I = TAN⁻¹ .21618 = 12° 11' 54"

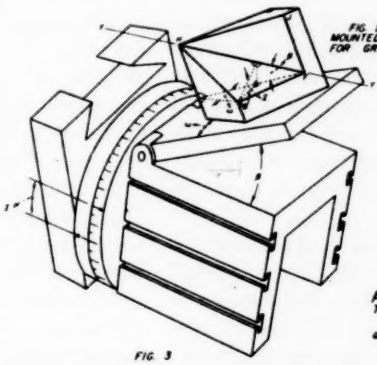

FIG. 3

IN ORDER TO GRIND THIS SURFACE, THE LINE YY MUST BE PARALLEL TO THE PATH OF THE GRINDING WHEEL, HENCE THE FIXTURE MUST BE TILTED AND ROTATED β' AND ω' DEGREES RESPECTIVELY.

THE FOLLOWING FORMULA MAY BE USED TO COMPUTE THE ANGLE OF ROTATION (ω').

β' = 25° (FIG. 1)
TAN ω' = TAN 30° COS β' =
.57735 ∿ .90631 = .52325
ω' = TAN⁻¹ .52325 = 27° 37' 14"

FIGURES 4 AND 5 SHOW SYMMETRICALLY OPPOSITE SECTIONS MOUNTED ON A DOUBLE TILTING DEVICE WHICH NECESSITATES DIFFERENT TILTING ANGLES FOR MACHINING SYMMETRICAL SURFACES.

SOLUTION FORMULAS

FIG. 4

ω = 30° (FIG. 1)
TAN β = TAN 25° COS 30°
= .4663 ∿ .86603 = .40383
β = TAN⁻¹ .40383 = 21° 59' 24"
TAN I = SIN β TAN 30°
= .37445 ∿ .57735 = .21618
I = TAN⁻¹ .21618 = 12° 11' 54"

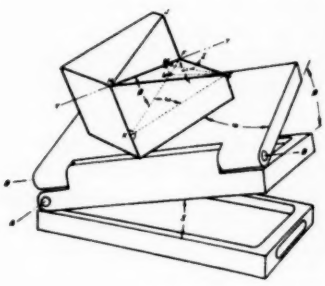

FIG. 4

FIG. 5

ω = 30° (FIG. 1)
TAN ∠DFE = TAN 25° COS 30°
= .4663 ∿ .86603 = .40383
∠DFE = TAN⁻¹ .40383 = 21° 59' 24"
TAN β' = TAN ∠DFE TAN 30°
= .40383 ∿ .57735 = .23315
β' = TAN⁻¹ .23315 = 13° 7' 27"
TAN I' = SIN β' COT 30°
= .22706 ∿ 1.7320 = .39326
I' = TAN⁻¹ .39326 = 21° 29' 4"

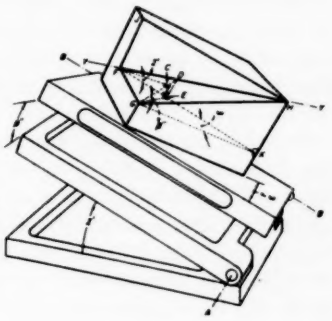

FIG. 5

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PLATE II

the tube delivers the maximum current. By throwing these voltages out of phase we can reduce the current delivered by the tube and otherwise alter its characteristics to meet specific welding needs. In the diagram E_s is the voltage through the secondary of the transformer, E_c the voltage through a capacitor and E_r the voltage through a variable resistance. The voltages may be changed and the voltage for the grid thrown out of phase. This reduces

(*Continued on page 27*)

Mathematics in Engineering*

By Robert Schilling

Research Laboratories Division, General Motors Corporation, Detroit, Michigan

You have heard and read so much about the urgent need for more scientists, engineers, and mathematicians in our country that I do not need to add to the discussion of the present and future shortage. I am sure all of you want to help in educating more young people for these professions by increasing the interest of your students in mathematics. What you need most is not more statistics but material that you can use in explaining just how mathematics is used in industry and why it is becoming more important with every passing year. Such material I want to present to you here.

The history of all our modern sciences and of engineering is the same in this respect: they began by observation of facts, and only after an accumulation of data did men start to analyze the facts. Purely experimental work leads to the observation of qualitative relations. Computation and predictions can be attempted only when we know what factors act in a process and what the nature of these factors is.

Examples of this sequence are numerous. Take possibly the best known case: Newton's formulation of the laws of mechanics. The motions of the stars and planets had been observed for thousands of years, but nobody had been able to define the basic quantities and their relations so that accurate predictions could be made. Men like Galileo had contributed experiments in simple terrestrial mechanics and had come close to a correct statement, but only Newton's definition of the basic units and of their relations made computation possible. And although he contributed immensely to the advancement of mathematics itself, his laws of mechanics are stated in terms of mathematics that had been known for a long time. You know how rapidly the use of mathematics increased from then on in the study of problems of celestial and engineering mechanics.

In the field of engineering, the story is the same. We accumulate more and more experience, a first-class mind defines the basic factors and their influence, and hundreds or thousands of others begin to compute useful data. The sequence is never finished, because as we understand simple processes, we are baffled by more complicated ones, and we need more discovery and analysis. During this development, more and more processes become accessible to computation and our need for able mathematicians increases constantly.

One would think that all engineering problems could very well be solved by application of enough analytical effort, because engineering consists simply in the control of physical and chemical processes whose laws are precisely formulated. But that is far from true. Engineering often deals with such a complex mixture and sequence of processes that the human mind cannot conceive and state them all in proper order, and we must select only the most important ones and follow their action. That means, of course, that we must compare observation and analysis until we find a hypothesis which gives correct results and which we can handle mathematically.

Take as an example the cycle of compression, ignition, and expansion in an automobile engine. The basic laws of thermodynamics were known long ago, and mathematical formulations for power generation cycles are well established.

* Presented at the 1952 Summer Meeting of the National Council of Teachers of Mathematics, Detroit, Michigan, June 30, 1952.

They have been used successfully in designing steam engines, steam turbines, and for refrigeration machinery, which runs this process in reverse. But they work only approximately in internal combustion engines. The reason for this poor showing is that we cannot state precisely just how and when the heat of combustion is liberated. The complicated mixtures of complex chemicals do not burn all at once or at the same rate and the combustion is very much affected by minor variations; we simply do not have enough information to write out the equations needed for a mathematical analysis.

In problems of mechanics, we are usually better off. We can always find mathematical formulations for the basic mechanical processes and can also describe processes so complicated and including so many variables that they completely confuse the unaided human mind.

I want to illustrate this situation by showing you a number of examples from the field of ride and suspension engineering for automobiles. We shall start with the simple algebra problems and look at more and more complicated ones until we come to what is at present the end of the road.

Since suspension is effected by springs, we must first describe the fundamental factors of springs and their action in terms of mathematics. Figure 1 shows the units and their relations for a coil spring; any other type of spring is subject to the same equations.

The spring has a free height when unloaded, which decreases when a load or force is applied. In first approximation, the load L is directly proportional to deflection D as shown in the equation, and we define the proportionality factor as a load rate or simply "rate" K. If we plot the load versus the deflection (as in Fig. 1), the load-deflection relation is represented by a straight line which has the slope K.

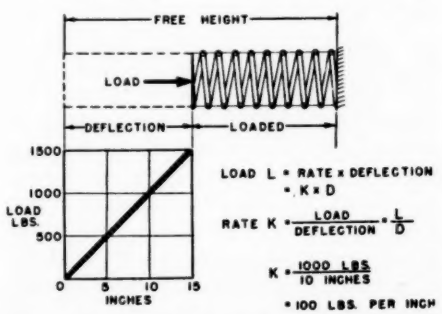

FIG. 1. Load—Deflection relations of a spring.

Note the similarity between Newton's second law:

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

and our spring equation:

$$\text{Load} = \text{Rate} \times \text{Deflection}.$$

It may seem presumptuous to set up such a simple engineering equation next to one of the most important equations of science, but the comparison can be used to show just how we must start in the application of mathematics.

An abstract, dry definition simply says that mass is the proportionality factor relating force to acceleration, and spring rate is the proportionality factor relating force to deflection. But the two equations do more than that. They accurately define a quantity for which we have an instinctive feeling and which helps us greatly in forming a clear mental picture of the action in a process. Everyone who handles a spring has the concept of "stiffness" and will conclude that this stiffness is important in determining the action of the spring, but quantitative analysis cannot start until we define stiffness or rate in such a way that its numerical relations to other factors can be stated.

We can go further and say that even qualitative thinking about these processes is confused until the factors are defined by some mathematical relation to others. There are many examples in science and engineering where equations are written out without any intention of using them for numerical calculation. The symbolism of mathematics leads to the clearest presentation of definitions, hypotheses, and conclusions.

Now let us try some simple application of our spring equation. Assume we have a

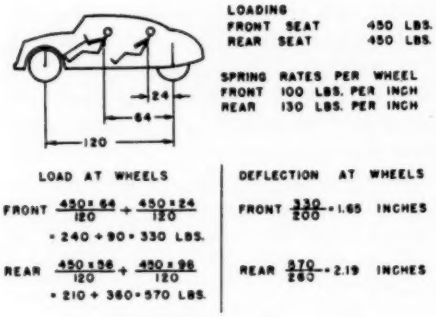

FIG. 2. Deflection of suspension due to passenger load.

car with the dimension shown in Figure 2. How soft can we make the springs so that the load due to driver and passengers will not change the standing height excessively? Let us assume that the front wheel loads are carried by springs of 100 pounds per inch rate each, and the rear wheels by springs of 130 pounds per inch each, and that front and rear seat each carry a load of three people weighing 150 pounds.

By simple proportion, we find that the loads due to six people at the front end amounts to 330 pounds and at the rear end to 570 pounds. Dividing these loads by the rate of two wheels for each end according to the formula

$$\text{Deflection } D = \frac{\text{Load } L}{\text{Rate } K},$$

(as in Fig. 1), we find the deflection at the front as 1.65 inches and at the rear as 2.19 inches. These deflections are both fairly large, and the springs cannot be made much softer.

When a wheel rolls over an obstacle of a given height, the excess force produced is again given by the equation

$$\text{Load (or Force) } L = \text{Rate } K \times \text{Deflection } D.$$

Hence, if we want to make the force small, we should make the rate low. Figure 2 shows us that there is a limit to this softening unless we provide a mechanism which will equalize the standing height of the car.

There is another way in which the potential flexibility of suspensions is limited, and we will see it if we carry out a more difficult analysis. A weight on a spring forms a vibrating system. Figure 3 shows the simplest analogy for a car on a spring suspension. We have here a sprung mass or weight, represented by a slab of some heavy material, supported by two springs which rest on wheels. The whole assembly rolls over a wave in the road so that its equilibrium will be upset. Analysis of this action goes beyond high school mathematics but some of the important results can be stated in simple algebra. Considering each end independently of the other, we find that they each represent a system whose free frequency is given by the equation in Figure 3. With the weights and spring rates indicated, which are quite

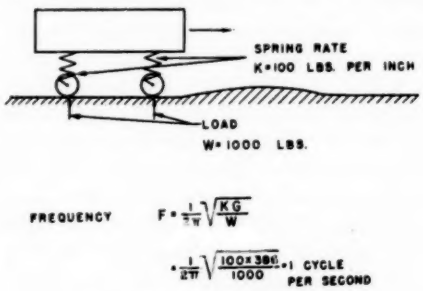

FIG. 3. Simplest analogue of vehicle and road wave.

representative, the frequency is just one cycle per second. This is a fairly slow motion which does not upset human beings unduly. Lower spring rates would slow down the motion still more and make for more comfort, but there is one difficulty.

A system with a frequency of one cycle per second returns to equilibrium at a certain rate, so that if the car comes to a change of road level, such as a ramp, it cannot line itself up instantly to this new level. The lower its frequency, the longer will it take for this readjustment. A car with the usual frequency characteristic is already too slow for some drivers, and they may not be happy if they have to slow down still more for a major change in the road.

Now let us see how the actions of front and rear combine when the car rolls over the road wave. In order to make this reasonably realistic, we must include the action of the shock absorbers, which damp out an initial displacement fairly rapidly. Figure 4 shows what happens to the two ends independently. The curves are computed by the well known equations of vibrating systems. The front wheel strikes the wave first and is already halfway up when the rear begins to lift. During the following descent, the front also leads the rear, producing a pitching motion of the car. The differences in displacement at any time represent the pitch angle, and by

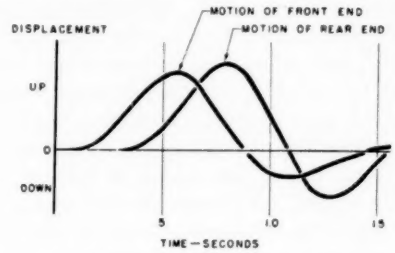

FIG. 4. Front and rear end displacement on a single road wave.

computing such curves for a number of road conditions, we can see just how a car would act.

The next diagram, Figure 5, shows the results of such a study. We see here that the short wave lengths produce more pitch, and that the pitching motion has resonance properties; starting at low car speed, the pitch on a given wave first increases with speed, reaches a maximum, and then decreases. This diagram shows the action of just one selected suspension arrangement. If we want to find the best combination of suspension elements, we must repeat the calculation many times for various arrangements and compare the results. You can well imagine that this takes a great deal of time, and in order to do it, we must use mechanical computation and employ engineers with considerable mathematical training.

But our analysis still does not represent

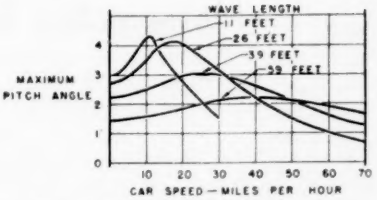

FIG. 5. Maximum pitch angles on various road waves.

reality well enough so we must complicate it even more. Figure 6 shows a refinement of our analogue over Figure 3. Here we have added the so-called unsprung masses, corresponding to the wheels and axles, and springs representing the tires. It is easy to see that the unsprung masses can move independently from the sprung mass and that they also represent bodies capable of vibration. Since we know the magnitude of the masses and all the spring rates, we can use the equation in Figure 3 and will find that the free frequency of the unsprung masses is about ten cycles per second or ten times as fast as that of the sprung mass.

With this information, we can again compute what happens when a wheel strikes a bump in the road. The results are shown in Figure 7 for a rather short wave. We see here that the motion of the sprung mass is fairly slow and has a small amplitude. That of the unsprung mass or wheel is faster and has a much larger amplitude. The forces transmitted to the

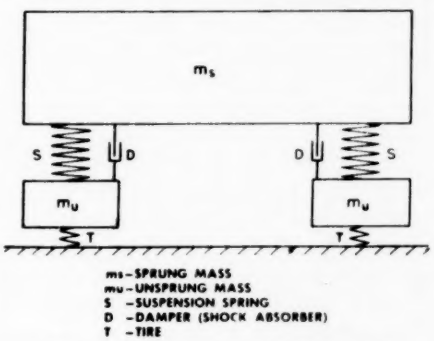

FIG. 6. Analogue of vehicle including unsprung masses.

(Continued on page 22)

Mathematics in a Retail Business*

By E. C. STEPHENSON
The J. L. Hudson Company, Detroit, Michigan

I HAVE LISTENED with great interest and respect to the discussions of Dr. Jacobson, Mr. Carlton and Mr. Schilling. Truly the adventures into higher mathematics to meet the engineering needs of industry, are of paramount importance to the creation of better products for the use of both producer and consumer.

I represent a department store, you know, one of those places which you rush into while on your lunch hour, and fuss because the salespeople seem so insufficient in number or so preoccupied in the discussion of last night's social activities that you cannot get the service necessary to satisfy your desires—and still have time to get lunch, and be back on the job to conduct your first after lunch class.

To discuss with you, the simple problems of a business such as ours seems almost a waste of your time, and yet, the problems need your attention.

The fact that our store is probably the largest store under one roof in the world makes our problem a question of degree but no different fundamentally than those of the smallest shop on any side street.

In the first 4 months of this year, February 1 through May 31 inclusive, we completed 10,500,000 individual sales transactions with customers. Since our return policy is extremely liberal we completed approximately 1,320,000 charge return and cash refund transactions during the same period. In addition, because of our policy, we completed over 30,000 policy adjustments with customers which did not result in the return of merchandise but did give to the affected customers, credit on their charge accounts or immediate cash in their hands to insure satisfaction in their purchases.

We have had a daily average of 33,000 salespeople on our selling floors during this period, a daily average of 53 people at service desks issuing charge credits and cash refunds, etc., and a daily average of 236 floormanagers who issue some charge credits and cash refunds in addition to their other duties.

Now each one of these 11,850,000 customer transactions required a simple problem in multiplication of quantity times price of article. Each one also required a calculation of 3% times merchandise value in order to determine and collect the amount of state sales tax. Probably 6 to 8 hundred thousand of them required the computation of the 20% Federal Excise Tax and a knowledge of percentages and about 750,000 of them required the computation and subtraction of discounts and a knowledge of percentages. Finally all of them required the determination of the net total of the transaction which requires the solution to problems of addition and subtraction.

In addition to all this, our salespeople operating cash registers made change *backwards* for about 4,200,000 of these 11,850,000 transactions. About 2,060,000 of these transactions required change making and careful money handling by a corps of 104 cashiers stationed on selling floors. Slightly over 600,000 of these transactions were delivered to the customer's home on a C.O.D. basis which required some activity in mathematics and careful money handling by our delivery men.

Then there are measure men who visit the customers to secure necessary information for making draperies and slipcovers, upholstering furniture, making venetian blinds, making mirrors, laying linoleum, making and laying carpets, installing metal and plastic wall tile, laying asphalt tile, making table pads and many other

* Presented at the 1952 Summer Meeting of the National Council of Teachers of Mathematics held at Detroit, Michigan, June 30, 1952.

activities. Also there are estimators converting all this information into the total charge to the customer for material and labor and submitting bids in order to secure the customer's approval of the transaction and authority to proceed with the work.

To this point the discussion has covered only the immediate contact with the customer. If at any point in the activities covered, arithmetic or other types of error occur we have an unhappy customer, and potential loss for the store. Every time the arithmetic is incorrect in favor of the customer—if the error is not caught before the transaction is completed the store is out of pocket. Losses of this character show up usually in the unexplainable part of our inventory shrinkages. If the error is in favor of the store, the customer usually catches it, and we not only have an adjustment to make to correct it, we have also an offended customer to pacify so that we may retain her good will.

Behind the scenes the problems are equally serious. The entire mass of sales and return data must be arranged so that we can determine cash accountability for salespeople, sales cashiering and service desk personnel and to determine cash differences for each. It must be arranged so that we know sales production for each salesperson, sales by departments, sales by classification or subdivision of a department, sales by kind of sale such as cash, charge, C.O.D., lay-a-way, installment; returns by these same breakdowns; charge sales and charge returns by alphabetic subdivision for the purpose of controlling charge business and preparing customers' statements. I should add that we have about 250,000 thirty-day charge accounts on our books.

Still further behind the scenes are the problems involving the analysis of the merchandise we sell so that we know what to buy, when to buy and and when to stop buying. At all times we are offering for sale some 4 to 5 hundred thousand different items of merchandise. These items are not always the same throughout the year, the different seasons cause a constant shifting of many, many, thousands of items. Still we must know when the demand will start, when it accelerates, when it plateaus and when it declines for each of them. We must know what prices our customers are willing to pay and what quality they are interested in.

The net result of all this is that we have large staffs of people analyzing our sales by supplier, by style, by material, color, size, price and various combinations of these items in order to be as intelligent as possible in our merchandising techniques so that we will minimize our selection errors and our losses.

To prepare our pricing charts under the applicable rules of the O.P.S., after the examination of two years invoices (and we receive about two thousand each day), and the selection of only one item at the same net cost after the application of cash discount, for each of the net costs at which merchandise is purchased for each department—we wound up with 52 volumes of about 75 pages each with an average of about 25 items to each page or about 98,000 items. For each one of these items it was necessary to list cost and selling price and to compute and list the markon percentages. On the basis of these charts we must compute the proposed selling price of new incoming merchandise by applying the markup per cent applicable to the net cost after deducting cash discount, of the item shown on the supplier's invoice.

I will not attempt to tell about all the arithmetic involved in central office payroll, statistical, cashiering, commission determination (every salesperson earns a commission which is paid weekly) accounts payable, accounts receivable and other office departments. The importance of this story is that the business of retail distribution is a vast problem of relatively simple arithmetic. Retailing needs thousands of people who are "good" at arithmetic, i.e., grade school and high school

arithmetic; people who have a knowledge of addition, subtraction, multiplication, and division; and who have the application of simple percentages so firmly ingrained into their thinking that the use of these fundamental arithmetic principles is almost instinctive.

Perhaps we do not use the proper skill to select people with this arithmetic knowledge. Perhaps mistakenly we believe that in an organization our size (employee staff ranging from 10 to 15 thousand depending on the season of the year) we have a fairly representative sample of the Detroit population and its scholastic accomplishments, but we assure you that our people are strangely lacking in a knowledge of arithmetic and writing legibly. From the time we were babies and received our first penny which we took to a neighboring store to buy a treat, all of us have been living in a world governed by simple problems in arithmetic. I have never been able to understand what lack in our schooling methods has permitted this fundamental skill to so deteriorate. In my humble opinion, somewhere in the school program we must change teaching methods in order to impress on our youngsters the necessity for a thorough understanding, knowledge and skill in the use of simple arithmetic.

Our hiring methods for the major segments of the organization do not permit of the use of testing methods as a prerequisite to an employment interview. The seasonal demand for people is sudden and requires a quick acquisition of workers and a quick termination of employment in order to keep operating costs in line. All selling and cashiering employees are sent through a training program lasting from three to five days, principally for the purpose of teaching a knowledge of the forms these people must be familiar with, and the many variations the sales data can take in its recording on the forms.

However, we do have a testing program for employment in the financial division of the business. This division employs from 800 to 1100 people depending on the season of the year. In February 1948 Dr. Bellows of Wayne University set up a testing program for our financial division. Using the Science Research Associates self scoring tests, which we still use, he determined passing scores on a test of 150 of then employees, 10% of whom we had previously rated unsatisfactory, 20% below average, 40% average, 20% above average and 10% superior.

The finance division does not test when it hires for Christmas peak staff, because there is insufficient time and too many failures would result if testing would be a prerequisite to hiring two hundred office workers in about 3 weeks time to meet this peak need. For the balance of the year testing is a prerequisite to an interview. We test for simple arithmetic, word selection, checking (relating one thing to another) and manual dexterity. Our program started in February 1948. We attempted at first to hire those who passed the tests at a 70 percentile ranking or better based on the scores of the guinea pig group. We found out after a short while that if we attempted to maintain such a standard we would get no people, so we dropped to 60 percentile or better.

Since starting we have tested 10,000 individuals who were applying for office jobs, and who expressed a preference for office employment at the prescreening section of our employment office; 3600 of them passed the test after dropping the requirements to a 60 percentile ranking. For one reason or another 1700 of the 3600 were not acceptable and we hired 1900 of those who passed the tests. During the war years we hired workers if they were still warm and looked as if they wanted work and our passing scores were based on the performance of people who came into our employment during those years. This record to my mind points up the lack of simple but fundamentally essential knowledge of those who have been graduating from our schools in recent years.

All this seems to be a far cry from the

implication of the impact on future methods of the analog and digital computers that Dr. Jacobson is spearheading in this important industrial area. However, it really has a close correlation to what he is doing. Because we have such a tremendous problem of arithmetic, analysis, and memory or storage of data, our store and a group of 23 other important stores scattered from coast to coast and from our most northern boundary to Florida are engaged in research to determine whether or not electronics can permit us at the point and moment of customer contact to complete all of our financial and statistical data concerning sales volumes and at the same time reduce the time required of the customer, after she has completed her selection, while our sales staff documents the sale. By required statistical and financial data, I mean

Dollar sales by department
Dollar sales by sub-department or classification
Dollar sales by department by kind of sale—cash, charge, installment, C.O.D., lay-a-way, etc.
Dollar sales by salesperson
Cash accountability by cash handler on selling floor
Dollar sales of labor or workroom services by department
Transportation costs collected from customer
Employee and other discounts allowed
Dollar amount of sales exempt from sales tax by reason for exemption
Sales tax collected from customer
Federal Excise tax collected from customer by selling dept. in which collected.

We should like the electronic device on the sales floor to accept input either from manual depression of keys or from an electric eye reading of holes punched into price tickets at the same time the data are printed on by the marking machine. We should like computers within the system to do all needed arithmetic problems for each sales transaction, of quantity times price—figuring the amount of discounts allowed, figuring state sales tax, Federal Excise Tax, and doing the needed additions and subtractions to arrive at net amount of the sale. We want this sales floor device to print out on the sales document all these data so that they are intelligible to the customer and the salesperson. Ultimately while all this is happening we want our accounts receivable posted so the customer's account is adjusted each time a transaction occurs that affects it.

This is not all we want done at the point of customer contact. While the electronics are doing all of the above financial problems, simultaneously we want them to analyze our sales by units sold about as follows: 1) we want number of units sold at each price by each sub-department classification; and 2) we want to know how many units are sold by resource from which obtained, style, size, color, material from which constructed, and price either in complete detail or in any desired combination of such factors required for the proper merchandising of the department involved. We want perpetual inventory kept where required.

All of this is not just a dream, a fantasy to be hoped for in the future. Part of it is here today. The J. L. Hudson Company has at this time the first electronic bookkeeping machine in America. It is a hand built device designed to carry memories on 6 kinds of related information for 10,000 items of merchandise. It is designed to do perpetual inventory. It permits access to information concerning any item of merchandise either at random or consecutively. It deal in units only.

A committee of which I am a member, studying this problem for the 24 stores mentioned, has on order at the present time a machine to do financial and merchandise statistical analysis including maintenance of perpetual inventory compilations, for a single department in a store. It is a point of *"customer contact"* recorder. It will not do all of the things we want accomplished, but it will do most of them. When it is built it will be installed in one of the stores for experimental and research purposes. Before we embark on store-wide systems we must be sure our progress is fool-proof.

The Radio Corporation of America announced to its radio and television dealers in its May Monthly *Newsletter* to such dealers, that it was entering the electronic record keeping field and that electronic methods would be particularly applicable to the problems of sales analysis, accounts receivable, accounts payable and perpetual inventory problems of retailers. However, electronic solutions to retailer's problems are still 5 to 10 years away. For the immediate future we must rely on school graduates to solve our problems in arithmetic. I hope that you and your fellow educators can reindoctrinate those now in school with the determination to become truly proficient in this essential art.

Mathematics in Engineering

(*Continued from page 17*)

body of the car are also shown and it can be seen that they combine the effects of both motions. Since all of you ride in automobiles, you can judge for yourself how well this picture corresponds to reality.

This is about as far as I can go along in the available time. Computations have been made which include more complications, but some results no longer check the observations satisfactorily. We are getting into the area where we have to know quite accurately what the shock absorber forces are when the wheel motion changes rapidly and irregularly, and what forces are acting between tire and road, not only vertically but also laterally and longitudinally, when the car rolls over uneven road surfaces. There is a lot more work to do, and when your present students start their engineering careers, they will find many challenging unsolved problems.

In the meantime, we can only encourage the boys and also the girls to go out for careers in science and engineering. We should therefore give them some good reasons to study mathematics as early as possible, and I have drawn up a few arguments.

1. Industry will do more and more analytical and computational work, and needs more people able to do it.

2. As new problems become accessible to computation, their complexity increases so that we must use more complex mathematics. We need not only more but better

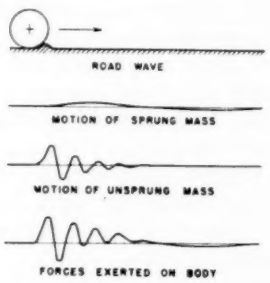

FIG. 7. Ride motion on a short road wave.

trained engineers and mathematicians.

3. Even for experimental engineers, a mathematics education in high school is an essential preparation to a college education. This is so, not just because of college entrance requirements, but because mathematics is the perfect introduction to the approach in science and engineering. All modern science deals with quantities and with numbers, and mathematics is the only way we know to reason quantitatively.

4. Mathematics education in high school is needed to arouse interest in science and engineering as professions. Very few students would choose such a career if they were not exposed to some training and practice in mathematics. It has been said that only a small percentage of all high school students will and can choose careers in science and engineering, and that therefore mathematics curricula should be limited. But the nation badly needs more technically trained people, and the high schools must help to steer young people into these professions.

The Need for Mathematics in Business and Industry*

A PANEL DISCUSSION

A. P. FUGILL, *Assistant Chief Electrical Engineer, The Detroit Edison Company*
W. H. HULSWIT, *Manager, Tire Engineering Research, U. S. Rubber Company*
C. V. WINDER, *Research Pharmacologist, Parke, Davis and Company*

A. P. FUGILL: The main purpose of this session is to correlate more closely the teaching of mathematics in the high school with the use of mathematics in everyday living. It should be recognized at the start that the high schools can and should teach only the fundamentals of mathematics. It is the function of industry to develop special techniques of use and facility in handling mathematics. The usual high school subjects, arithmetic, algebra, geometry, and trigonometry, are useful in varying degrees in the practice of engineering, or for that matter in any industrial and commercial activity. It would be wasteful to bring every student up to the degree of proficiency required in a few specialized fields. The effort in high school teaching, therefore, should be to use mathematics as a means of developing logical reasoning and judgment, rather than as an exercise in memory.

It has long been recognized that to be a good engineer, a student should be able to handle mathematics with facility. In mathematics courses as taught in school, however, a good memory may substitute for basic reasoning, so that a student can make good grades in these subjects and still not be a good prospect for engineering. In student guidance work, it is essential to recognize this situation. Although technological personnel of all sorts are badly needed, it is a grave error to steer young people into these fields unless they are going to be satisfied with the work.

In the field of electrical engineering, knowledge of the basic mathematics, at least through algebra, is essential. A large percentage of those practicing engineering today do not require advanced courses in mathematics, although some knowledge of geometry and trigonometry is useful. The need for advanced knowledge will be increasingly more necessary in the future, however, because of the scarcity of trained engineers. More and more, much of the work now done by engineers will be transferred to technicians in order to use the specialized knowledge of the engineer to the best advantage. Gradually, the man with the greater knowledge of advanced mathematical techniques will tend to emerge from his less fortunate fellows. The increasing use of large scale computing machines will accelerate this trend. Certainly, every effort should be made at the high school level to increase the number of students taking mathematics, as far more mathematically inclined and trained people will be needed throughout industry. Many of these will be in the technician field where, perhaps, the only real training they will get in mathematics comes at the high school level. Although the broadening influence of social and other generalized subjects in high school is valuable, it should not be obtained by sacrificing the training in mathematics.

W. H. HULSWIT: In the few minutes that have been allotted to me I cannot possibly cover the multitude of ways in which mathematics, from simple arithmetic to non-linear partial differential equations is being used in Tire Development work. I am going to content myself with a short

* Presented at the 1952 Summer Meeting of the National Council of Teachers of Mathematics held at Detroit, Michigan, June 30, 1952.

discussion of a complex problem for which we still do not have an entirely satisfactory solution.

You are all familiar with pneumatic tires on your automobiles. You probably know that they are made from natural and synthetic rubber and textile material such as cotton or rayon. The tire is fabricated while the rubber is the form of a very viscous liquid. Then, through vulcanization, the rubber is transformed into an elastic solid. This vulcanization is brought about by heating the rubber article. For example, the cure on a passenger tire might be 40 minutes at 300° F.

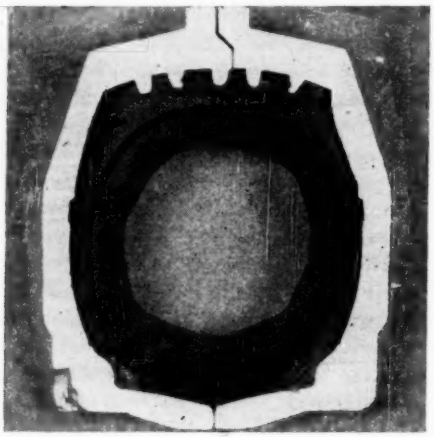

FIG. 1

With this introduction, let us look at Figure 1. Here we see the cross section of a tire mold with tire and curing bag in place. The outside of the mold might be heated by circulating steam to 300° F. and the inside of the curing bag might be heated by circulating hot water at 300° F. and 300 p.s.i. pressure. The temperature of a point near the surface follows very closely the mold temperature while interior points may have a considerable temperature lag as can be noted in Figure 2.

We would like a simple functional relationship connecting temperature and time at any point in the tire so that for any arbitrary internal and external temperature cycle the cure throughout the tire could be readily computed. The partial differential equation is easily written down. The transformation into toroidal coordinates is complex but available in books. The boundary conditions are extremely difficult to apply and are different for nearly every kind of tire we make and we have more than 3000 active size and brand combinations. The general analytical solution is therefore so formidable that it has never been seriously attempted.

Electrical analogues are a possible way to solve this thermal problem and some progress has been made in this direction. Generally such equipment is limited to a two dimensional study of the problem, which somewhat narrows its utility. Thus, most of our knowledge is the result of direct measurement with thermocouples built into tires. These data are expensively and slowly obtained and never in sufficient quantity to thoroughly cover the field.

It is not at all uncommon in dealing with practical industrial problems that exact analytical solutions have never been achieved and at best we have working approximations sometimes not very exact. Industry needs more and better practical mathematicians to solve their problems.

C. V. WINDER: My work is the study of the actions of experimental drugs on living processes in animals.

I apparently provide the contrast on this panel. Unlike the mathematically sophisticated people about me, I'm a mere biologist. Everyone knows that biology, rather than engineering, for example, is liable to be taken up by the non-mathematically inclined person. I deliberately use the word liable rather than likely. To the advantage of biology, however, the subject is taken up by many who are mathematically inclined also. But consider, for a moment the case of the person without mathematical inclination who takes up biology. Fortunately for him, to the present, biology is a vast field still containing substantially unexplored areas. This means that the person whose train-

ing is mainly in gross verbal description may still find an area from which he can hew out an important contribution. It may, perhaps, be mainly a scientific contribution with literary content, or mainly a literary contribution with scientific content.

Now consider that another kind of person might enter this same virgin field of exploration. This person has been able to say that for such reasons this person is likely to do a more forceful job of qualitative description than the other person, whether the emphasis be scientific or literary, and whether the field be biology, or, for example, politics. Actual recollection and use of technical mathematical details is unimportant here; what counts is the influence of their pleasurable past use on general manner of thinking.

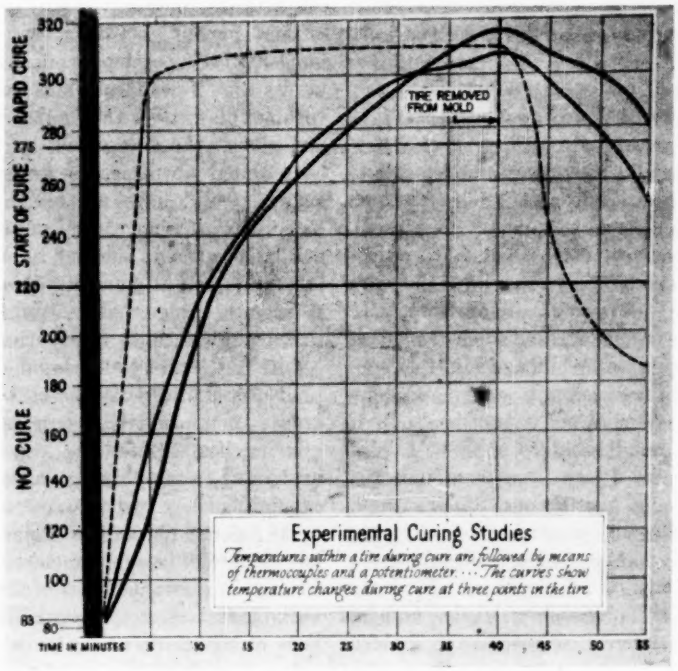
RAPID CURE
START OF CURE
NO CURE
TIRE REMOVED FROM MOLD
Experimental Curing Studies
Temperatures within a tire during cure are followed by means of thermocouples and a potentiometer. . . . The curves show temperature changes during cure at three points in the tire
TIME IN MINUTES

FIG. 2

enjoy a give and take experience with algebra, geometry, trigonometry, analytic geometry and other mathematics courses. The pleasure of this experience may have stemmed mainly from his own make-up, mainly from an exceptional teacher's clarity of presentation, or importantly from both. In any case we shall describe him as mathematically inclined. He has an added sense of logical orderliness that has come from a period of living with algebra and geometry. He has a feeling for symbolic representation of complex concepts that defy description in ordinary words. I dare

More important, perhaps, than the influence of mathematics on the qualitative descriptive phase of a field of endeavor, is its influence on the person's ability to carry the work beyond that phase. The person who has known some mathematics is likely to be interested in counting, measuring, and weighing, not as ends in themselves, but as preliminary steps to the solution of inter-relationships in the subject matter under investigation. Even to know the existence of qualitative interrelations requires quantitative technics. Individual counts or measurements in biological ma-

terial vary over wide ranges. One may, I think, come to feel that Nature at her very roots is statistical in structure and phenomenology—that she may be regarded as a mysteriously complex universe of distributed populations—populations of things and populations of events. Horizontally, populations are endlessly correlated, while vertically they actually make up populations of larger units and in turn are made up of populations of smaller units. Approaching Nature with the philosophy and technics of statistics may be considered a sympathetic approach.

Thus, quantitative, and even qualitative description and elucidation of interrelationships in the subject-material bring to mind the usefulness of such technical concepts as certain kinds of population distributions for which workable mathematical models exist—the binomial distribution, the normal distribution, the Poisson distribution—and such things as the meaning and technics of multiple correlation and regression. It is soon apparent that whole populations cannot be studied. Relatively small samples must be taken, and inferences drawn from them concerning the parent populations. Small sample technics become essential. Such concepts as sample mean, estimate of standard deviation, degrees of freedom, Student-Fisher's "t," probability levels, fiducial limits, Chi-square, etc., are to the modern biologist as the hand-wrench is to the mechanic. Thus, the ability with which the person carries forward the quantitative descriptive phase of investigation depends heavily on his arithmetical acumen and general understanding of the working models provided by mathematical statisticians. Believe me, he may well even wish to be sufficiently adept at the mathematical aspect to create new models that his work provides a need for.

Before qualitative and quantitative description are far along, some one will enter the field with a particularly mischievous type of curiosity. He is known as an experimenter. Likely he is a person who has enjoyed the manipulations of symbolic logic in mathematics. He will deliberately do unnatural and unfair things to the subject material to observe whether or not certain expected or unexpected changes occur in the distributions of things and events. He may call this the testing of hypotheses. It will represent an attempt to uncover things and relationships that were not apparent or certain from mere observation.

This person, however, cannot allow himself to be completely mischievous. He knows that if he does things and watches without plan, then the changes in things and events that occur will be hopelessly confounded outcomes of tinkerings and chance. He knows of the Englishman, Ronald A. Fisher, both mathematician and biologist, who outlined in bold strokes the fundamental concepts of modern experimental design and analysis. He knows that these concepts of experimental procedure are special developments of the kind of statistics that the *observer* had to know. He knows that there is a rapidly growing list of reference and text-books explaining, exemplifying, adapting, and expanding these Fisherian methods.

He knows that in the realm of experimental design lie such matters as the need for active randomization of uncontrolled variable factors, so that assumptions at the base of the mathematical models of subsequent analysis are satisfied. He knows how certain other variable factors can be controlled by design, and how still others can be controlled statistically through the adaptation of regression technic known as analysis of covariance. He knows how, with various degrees of certainty—never complete—to make qualitative inferences as to relation of outcome to procedure. He knows how to make quantitative statements that a relationship is between such and such numerical limits, with given odds of being correct.

What is the point of all this?

First, a thought certainly not new to this group; namely, that even for qualita-

tive work, whatever the field of endeavor, at least a brief, early period of happy marriage with conventional elementary mathematics is an important phase of intellectual development.

Second, beyond the most elemental phase of development of a science, particularly a biological science, the philosophy and technics provided by mathematical statisticians have rapidly become requisite. Statistics must now be considered an integral part of science.

Hence, to the extent that science is introduced in high school, so, too, and integrated with it, must be introduced some notion of statistics.

Mathematics in the Skilled Trades

(*Continued from page 13*)

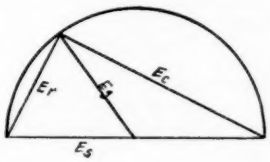

FIG. 9

to a very simple problem in which our old acquaintance, a right angle inscribed in a semicircle, enters. The diameter is the secondary voltage, the legs of the angle are the voltages through the capacitor and resistance and the radius is the grid voltage. Its angle with the diameter shows how much it is out of phase. Our problem is thus reduced to a mathematics problem which can be easily handled. This is illustrative of the powerful assistance mathematics renders to the skilled worker in the field of electricity and electronics.

We have included problems in our demonstration which indicate the need for considerable power in mathematical analysis and problem solving ability. We cannot expect high school students to come into industry equipped to handle these problems. We do feel, however that they should have achieved sufficient mastery of mathematics to enable them to master problems of this type without too much difficulty. The student who masters mathematics will greatly help to qualify himself for a career in the skilled trades. Any one of the skilled trades will offer job opportunities in great numbers for many years to come in fields of great interest and variety and which offer splendid opportunities for advancement. Therefore the mathematics teacher has something to offer of great value and should counsel and guide students into these classes. The few examples pointed out here will perhaps suggest avenues of motivation helpful in stimulating greater student interest.

HAVE YOU SEEN?

In *Scripta Mathematica*, June 1952
- "The Mathematician's Obligation to the Engineer" by Arnold Dresden
- "A Brief History of the Investigations on Mersenne Numbers and the Latest Immense Primes" by Horace S. Uhler
- "A Theorem on Divisibility" by H. Grünbaum
- "The Area of a Triangle as a Function of Its Sides" by Victor Thebault
- "A Method of Squaring Large Numbers by Means of Number Triangles" by J. Ginsburg
- "Pythagorean Triangles form Fibonacci Numbers" by Harlan L. Umansky
- "A Nest of Identities" by Royal V. Heath
- "Lucky Strikes" by Alfred Moessner
- "Is Homework Really Necessary? (With Special Reference to Mathematics)" by William D. Reeve

In *The Journal of Engineering Education*, September 1952
- "A Goal for American Engineering Education" by S. C. Hollister
- "Educating Engineers for 2000 A.D." by C. C. Furnas
- "Improvement of Engineering Teaching" by the Committee on Improvement of Teaching

REFERENCES FOR MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

Notes on Analytic Geometry

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NOTE 1. An excellent reference work dealing with plane curves is Robert C. Yates: *A Handbook on Curves and Their Properties*, Ann Arbor, Michigan, Edwards Bros., 1947; 245 p.

NOTE 2. For references dealing with visual aids in analytic geometry, models of conic sections, the mechanical construction of curves, etc., consult the Mathematics Teacher, October 1951, vol. 44, pp. 424–25, 428; also, *National Council Teachers of Mathematics*, 18th Yearbook, 1945, pp. 223–25, 422–28.

The fact is that **there are few more "popular" subjects than mathematics.** Most people have some appreciation of mathematics, just as most people can enjoy a pleasant tune; and there are probably more people really interested in mathematics than in music.

—G. H. Hardy, *A Mathematician's Apology*, 1940, p. 26.

The President's Page

NEW YEAR RESOLUTIONS

AT THE beginning of a new year many people feel an urge to make resolutions directed toward the improvement of their chosen life work. Such a practice seems a highly desirable one particularly if the urge is sufficiently strong to be accompanied by the determination to hold fast to the carrying out of the resolutions. By the time that members of the National Council read these paragraphs I hope that self-evaluation of 1952 has already suggested some worthy objectives for special new emphasis in 1953.

In this connection, I would like to make a suggestion and to illustrate it. We all recognize that the importance of our work, that of teaching mathematics, will be even greater in 1953. A formulation of our individual convictions about the underlying reasons for this importance, and more specifically a listing of objectives which are suggested by this importance, are desirable professional activities. A new resolution to so teach that our students and our communities know more certainly that we hold such convictions, and understand more clearly our objectives, seems to me highly appropriate. A statement of some of my convictions and objectives, upon which I have based a 1953 resolution, is a statement that was in part formulated earlier and which in its earlier version a number of the readers of this page may have heard. As an illustration of the suggestion that I want to make in this month's contribution this statement is given here:

I believe that there are common goals which all men should share: to make our earth a better place for children to grow and people to live, to contribute to the establishment of peace and good will among men, to make knowledge and understanding of man's achievements accessible to all.

I believe that as teachers we hold a unique responsibility and opportunity to help children understand the world in which they live, themselves, and man's great achievements; to help them grow in the power of observation and willingness to face facts; and to help them acquire the disposition and ability to seek in the world the best common goals.

I believe that as teachers of mathematics we have the responsibility and opportunity to help students really enjoy the study of mathematics, to help them know how this study leaves open the door for them to greater opportunities, to help them experience new ways of thinking and develop better thinking habits, to help them develop scholarly habits and appreciate the value of scholarship, to lead them to an appreciation of the role of mathematics in our culture and civilization, and to so teach that their study of mathematics provides a sense of personal achievement, and is in reality a personal achievement. Too, as teachers of mathematics we have an important opportunity to make knowledge and understanding of man's achievements in a special, but exceedingly important area, accessible to all. Furthermore, I am convinced that these opportunities lie inherently in the very nature of mathematics itself.

I believe also that it is necessary to hold firm to a faith that one's beliefs are worth working for.

For those who have not yet made their 1953 resolutions directed toward professional improvement or for those who may wish to give additional thought to possibilities for the new year, this suggestion with illustration has been given. Naturally the statements of others will be different, perhaps less idealistic and more specific, or the other way around. It is certainly not the intention to propose this statement as one suitable for others, for great value can come from just the attempt to formulate a statement of beliefs and/or objectives which are one's own. Most surely if they are our own, we have a much better chance of achieving them because we will work harder toward that end.

JOHN R. MAYOR
President

WATCH for the Program of the 31st Annual NCTM Meeting at the Hotel Ambassador in Atlantic City, April 8-11, 1953, in your February issue.

The Fourth Delegate Assembly

MARY C. ROGERS, *Chairman*
Committee on Affiliated Groups
Roosevelt Junior High School, Westfield, New Jersey

THE FOURTH DELEGATE ASSEMBLY of the National Council of Teachers of Mathematics will be held at The Ambassador in Atlantic City, New Jersey on Thursday and Friday, April 9 and 10, 1953. There will be a breakfast for delegates on Thursday, followed by a two-hour session of the Assembly. A second session will convene on Friday morning.

The National Council has gained tremendously in influence and effectiveness over the years. This growth is due in large part to the strength of the leaders who have directed its activities and programs of service. "The most outstanding persons in mathematics education have been leaders and participants in the work of the Council."* A very great deal of the strength of the Council is also due the local organizations of mathematics teachers. Their loyalty and enthusiastic support furnish the foundation upon which this great national organization is built.

It has been most interesting and inspiring to study the many letters to Dr. John Mayor from leaders of these local Affiliated Groups. The enthusiastic accounts of many activities being carried on—the frank statements of problems arising—the expressions of appreciation for the assistance received through affiliation—all indicate an outstandingly fine relationship.

This spirit of cooperation and mutual helpfulness has been increasingly apparent at the Delegate Assemblies, making these sessions an invaluable aid to the work and service of the Council. Not only is coordination of program and service between the Council and its Affiliated Groups of the highest order; but there is apparent from Assembly deliberations an equally inspiring evidence of most extensive coordination of local mathematics programs with the activities of many and diverse professional associations and other educational agencies throughout the country. It is with this spirit of cooperation in mind that we have planned the agenda for the Fourth Delegate Assembly around the central theme of *Cooperation and Coordinated Endeavor.*

The National Council has increasingly encouraged shared planning as well as wide-spread participation by its members in all of its programs and services. In keeping with this philosophy and practice, Dr. John Mayor—then Chairman of the Committee on Affiliated Groups—requested that the Third Delegate Assembly appoint a committee from its own personnel to assist with the details of policy and planning, and to aid in determining agenda for future sessions of the Delegate Assembly. The following committee was appointed: Daniel B. Lloyd, *Chairman*, Wilson Teachers College, Washington, D. C.; Mary Lee Foster, Henderson State Teachers College, Arkadelphia, Arkansas; W. V. Unruh, Shawnee Mission High School, Merriam, Kansas.

The agenda for the Fourth Delegate Assembly has been prepared from recommendations made by this Planning Committee supplemented by suggestions from the regional representatives for Affiliated Groups; by further suggestions from officers of Affiliated Groups and other members of the National Council.

We urge that prospective Delegates study these agenda carefully; that they also determine the point of view of the Group which they represent, so that our

* From the Publicity folder, "NCTM, The Professional Organization."

discussions at the Assembly may voice the opinions and advice of as large a segment of National Council membership as possible. We also invite all readers of THE MATHEMATICS TEACHER to write us their reactions to these plans, and to make whatever suggestions they believe will add to the strength and effectiveness of the Assembly deliberations.

Your Planning Committee suggests that some of the major topics here outlined be presented at the Assembly by a panel of speakers and then thrown open for general discussion. We invite your reactions to this plan. We shall be happy to receive your recommendations as to the personnel for these panels.

You will want to hear from our Regional Representatives—Dr. Donovan A. Johnson, Dr. William A. Gager, Mr. Jackson B. Adkins, and Miss Ida May Bernhard. We are giving them a very special place on the Assembly program. Many of our special services and agencies are directed by specialists in these areas. We shall invite these specialists to discuss their services with the Delegates at the Assembly and to solicit whatever assistance they believe will make their work a greater contributing factor toward the improvement of mathematics education for the youth of our country.

Topics proposed for discussion at the Fourth Delegate Assembly and on which the Delegates may wish to make recommendations to the Board of Directors are summarized through the following questions:

1. What are desirable functions and responsibilties of the Delegate Assembly as representative of the membership of the National Council and of the Affiliated Groups?
   a. How can the Delegate Assembly exert a greater reciprocal influence in the affairs of the Council?
   b. How can the Delegate Assembly better help the Council to grow in size, strength, and effective service?
2. What ways and means can be devised to improve the liaison between the field and the national organization for the purpose of:
   a. Publishing more news of improved practices and techniques.
   b. Obtaining speakers, leaders, and special talent for assistance to:
      1) Program Planning Committees
      2) The Yearbook Planning Committee
      3) THE MATHEMATICS TEACHER
      4) The Committee on Publications of Current Interest
      5) Other Service Groups within the National Council.
3. Do you find the Affiliated Groups *Newsletters* helpful? What suggestions have you for their improvement and more effective service? Should each issue report interests and activities of the entire Affiliated Groups Organization; or should each of the four geographical areas, into which the organization is now divided, take turns in reporting its activities through its own special issue of the *Newsletter*?
4. How can the Affiliated Groups be of greater assistance to State Representatives in the stimulation of membership growth, and in the dissemination of National Council publicity?
   a. Could each Affiliated Group help promote the work of National Council more effectively by having a Coordinating Committee to assist the State Representative with her duties?
   b. Can the collection of dues for National Council membership and for membership in the Affiliated Groups be better coordinated?
      1) What methods for such coordination are now being used?
      2) Should these methods be standardized throughout the country?
      3) How would this affect membership both locally and nationally?
      4) Should there be uniformity in the determination of the fiscal year among Affiliated Groups—in agreement with the fiscal year of National Council?
5. What are the duties of each officer of the National Council or of an Affiliated Group as a Public Relations representative for the National Council?
6. In what specific ways can affiliation assist local groups in effecting a closer coordination of their programs
   a. With the programs of other state and regional mathematics groups?
   b. With educational programs in other fields of learning?
   c. With the educational agencies of business and industry?
7. In what specific ways can the Committee on Affiliated Groups help you work out a plan of cooperation with other Groups in the matter of research and in the study of common problems?
8. What more needs to be done to promote and improve the special activities initiated through the Affiliated Groups program?

(*Continued on page 37*)

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

66. More About Quadratics

In Miscellanea 19, [THE MATHEMATICS TEACHER, vol. XLIV (March, 1951), p. 193], we published a list of methods for solving quadratic equations which was compiled by William J. Hazard. The following comments are from a later letter in which Professor Hazard clarified one or two of the items he had listed.

As an illustration of method 2 (factoring after a substitution, $z=ax$), and of method 3 (factoring in pairs by splitting bx into two terms), let's apply them to the example used by Mr. Fitz in THE MATHEMATICS TEACHER for December, 1950 (vol. XLIII, p. 421), $12x^2-11x-56=0$.

Following method 2, let $z=12x$ and the equation becomes after substitution and clearing of fractions $z^2-11z-(12\cdot 56)=0$. This equation can then be solved directly, or it can be used as as introduction to an explanation of method 3. Here we choose to do the latter. We seek two numbers whose product is $12\cdot 56$ and whose difference is 11.

$(12\cdot 56)=(4\cdot 3)(7\cdot 8)=(3\cdot 7)(4\cdot 8)=21\cdot 32$. This would lead immediately to the solution of the equation in z, but it also means that the original equation can be written:

$$12x^2-32x+21x-56=0$$

or

$$4x(3x-8)+7(3x-8)=0$$

or

$$(4x+7)(3x-8)=0.$$

The remainder of the solution is obvious. Please note that it is not necessary to do the substitution as a preliminary to the above factoring by "grouping," and hence methods 2 and 3 are different and distinct, though related.

Under the stress of war preparation in the early 40's, the instructors in all services were made to cut the corners in everything, no matter how slight. Their lectures consisted largely of "Do this and then do that." About 1942 or before, the Training Division, Bureau of Naval Personnel, Navy Department, issued a pamphlet of "Sample problems and examinations taken from the field of instruction in aviation and other naval subjects." Discussing the quadratic equation $ax^2+bx+c=0$, they state "When the coefficient of the first degree term is an even number, use the 'Half Formula'

$$x=\frac{\frac{-b}{2}\pm\sqrt{\left(\frac{b}{2}\right)^2-ac}}{a}$$

obtained by dividing numerator and denominator of the regular formula by 2."

Professor Fehr's very excellent article [see method 12 in *Mathematical Miscellanea 19* cited above] on the "Graphical Representation of Complex Roots," in the *Eighteenth Yearbook* of the National Council of Teachers of Mathematics, is the best discussion I have seen, and his recommendation of Figure 2 for the high school class is without question.

From the mechanical draftsman's standpoint, the tangent points R and R' in his Figure 1 are indeterminate, and so for accuracy the figure should be drawn in reverse order. Call the vertex of the parabola V and lay off $VQ=VA$, then draw $R'R$ through Q.

This may seem like making sure of a bullseye by shooting at the side of the barn and drawing the target around the bullet hole. Draftsmen are likely to do unorthodox things, but this contruction is justified by the algebraic relations also.

I have a letter from our historian Florian Cajori, dated Oct. 26, 1923, in answer to my call for help about Monsieur Lill [Prof. Hazard listed the "Lill circle" as his methods 12 and 13 in *Miscellanea 19*]. Cajori stated that Maurice d'Ocagne uses a peculiar method of approximating the roots of nth degree equations which he attributes to Lill, but he seems to assume that of course everybody knows about Lill, for he fails to enlighten us.

Cajori mentions his own work on the history of arithmetical methods of solution, published in "The Colorado College Publication, 1910," and gave me the following references to Lill's work:

Mehmke's article "Numerisches Rechnen" in the *Encyklopädie der Mathematischen Wissenschaften*, Erster Band, "1F" pp. 1012, 1013, 1014.

Lill's method for real roots was published in 1867 in the *Nouvelles Annales de Mathématiques*, 2nd series, Vol. 6, p. 359, and its extension to complex roots appeared in 1868, in the same publication, Vol. 7, p. 363. From these articles we learn that E. Lill was a captain of engineers serving in Austria.[1]

Cajori goes on to say "I made a start on graphic methods of solution, but found the field so very extensive that I gave up that historic study in despair. The number of papers on that subject is simply enormous." So if Cajori had to throw up his hands, where would a novice expect to get in? I'll just pass on by, without tapping on the door. With apologies to Solomon, "Of making many quadratic solutions, there is no end."

Sincerely yours,
WILLIAM J. HAZARD
1001—10th Street
Boulder, Colorado

67. Lill's Method for Evaluating Polynomials.

Lill's original publications on this subject dealt with the solution of rational integral equations. The method however can be regarded as a procedure for evaluating polynomials, with the solution of equations being regarded as merely the determination of an x for which the polynomial is zero.[2] For a second degree equation the method specializes to the "Lill Circle" method, as Professor Hazard named it, to be found in two of L. E. Dickson's works cited in *Mathematical Miscellanea 19.* For $x^3-2=0$ it specializes to a method for duplicating the cube which has been attributed to Plato.

Given any polynomial with real coefficients, $x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots a_{n-1}x+a_n=0$, the procedure is as follows: Take a convenient length UA_1 (Fig. 1) as a unit. This length may, then, also be regarded as corresponding to the coefficient of x^n. Perpendicular to UA_1 take $A_1A_2=a_1$ laid off to the right or left from UA_1 as a_1 is positive or negative. Take $A_2A_3=a_2$ at right angles to A_1A_2 and to the right or left from it as a_2 has the same or a different sign from a_1. Continue drawing right angled line segments until a_1, a_2, $\cdots$, a_{n-1}, a_n have all been represented.

Figure 1 corresponds to $P(x)=x^4+a_1x^3+a_2x+a_3x+a_4$, where $UA_1=1$, $A_1A_2=a_1$, $A_2A_3=a_2$, $A_3A_4=a_3$, $A_4A_5=a_4$. If now from

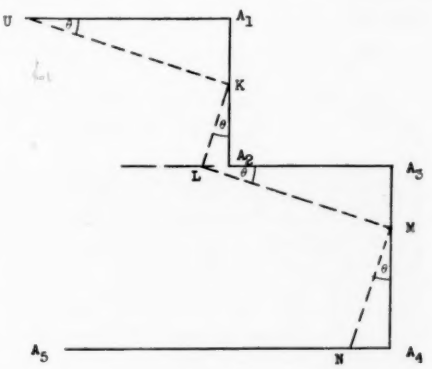

FIG. 1

U to A_1A_2 the line UK is drawn in the direction θ where

$$\theta=\tan^{-1}\frac{A_1K}{UA}=\tan^{-1}\frac{A_1K}{1}=\tan^{-1}A_1K,$$

KL is drawn at right angles to UK until it meets A_2A_3 in L, and this procedure is continued until the last dotted line meets the last line in the original rectangular figure (in this case MN meets A_4A_5 in N), then the remainder of the last segment (in this case NA_5) is equal in length to $P(\tan\theta)$.

To see this, first note that Figure 1 is drawn as if a_2, a_3 and a_4 were negative, and, further, θ is taken as negative so that $A_1K=\tan\theta$ must also be negative.

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[1] In addition to the two articles cited above from Cajori's letter to Professor Hazard it should be noted that an article by Lill appeared in *Comptes Rendus hebdomadaires des Séances de l'Académie des Sciences*, Paris: Tome 65 (1867), p. 854 ff. and that additional discussions appeared in Rudolf Mehmke, *Leitfaden zum graphischen Rechnen*, Leipzig und Wien: (1924), p. 22 ff., and in R. Mehmke, M. d'Ocagne "Calculs Numeriques," *Encyclopédie des Sciences Mathématiques Pures et Appliquées.* Tome 1, vol. 4 (1909), p. 336 ff.

See also the next item in this magazine, *Miscellanea 68.*—P.S.J.

[2] This approach is to be found on pages 27–30 of H. W. Turnbull's *Theory of Equations.* New York: Interscience Publishers, 1947. A similar treatment appears on page 287 of Volume I, Parte II of the *Enciclopedia delle Mathematiche Elementari e Complementi.* Milano: Ulrico Hoepli, 1950. Lill also noted this fact and the use of a circle in the special case of the quadratic equation in his second article on the subject; namely, that cited in the preceding footnote.

Then (in the drawing A_4A_5 represents a_4, a number less than 0, and $A_4N>0$)

$$\begin{aligned}NA_5 &= a_4+A_4N\\ &= a_4+\{a_3+A_3M\}\tan\theta\\ &= a_4+\{a_3+LA_3\tan\theta\}\tan\theta\\ &= a_4+\{a_3+[a_2+KA_2\tan\theta]\tan\theta\}\\ &\quad \tan\theta\\ &= a_4+\{a_3+[a_2+(a_1+1\cdot\tan\theta)\tan\theta]\\ &\quad \tan\theta\}\tan\theta\\ &= a_4+a_3\tan\theta+a_2\tan^2\theta+a_1\tan^3\theta\\ &\quad +\tan^4\theta\\ &= P(\tan\theta).\end{aligned}$$

or taking $x_1=\tan\theta$ and reversing the order of terms

$$P(x_1)=x_1^4+a_1x_1^3+a_2x_1^2+a_3x_1+a_4.$$

Any value of $x_1=\tan\theta=AK$ is a root of $P(x)=0$ if the dotted rectangular path

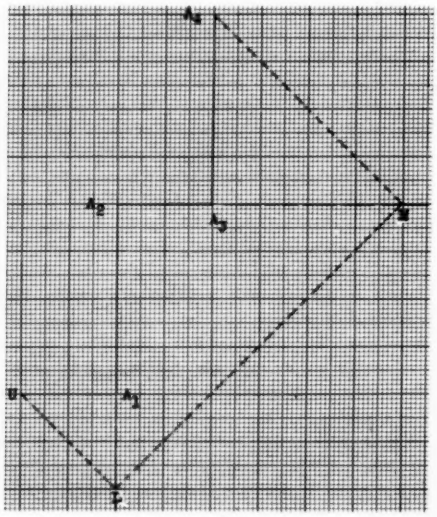

FIG. 2

which it generates ends with the last point of the original rectangular path as in Figure 2 which shows $x=-1$ to be a root of $x^3-2x^2-x+2=0$. Can you find the other two roots?

Figure 3 shows the Lill diagram for the quadratic equation $x^2-4x+3=0$. $OA_1=1$, $A_1A_2=4$, $A_2A_3=3$, $A_1R_1=1$, $A_1R_2=3$, and hence the roots are $x=+1$ and $x=+3$ since θ_1 and θ_2 are both positive.

The "Lill circle" construction for $x^2-bx+c=0$ ($b>0$, $c>0$) has been superimposed upon Figure 3. It consists of taking line A_1A_2 equal in length to b, drawing parallels A_1O and A_2A_3 equal in length to 1 and c respectively. Then contsructing the circle having OA_3 as a diameter. The intersections of this circle with A_1A_2 give the roots of the equation. The dotted lines and the fact that every angle inscribed in a semicircle is a right angle show that the two methods are equivalent. Easy modifications of these procedures take care of the cases in which the leading coefficient is not 1, some of the coefficients are zero, and the signs of the coefficients are different. The proof of the circle solution may also be arrived at either by assuming axes and applying analytic methods, or by noting that $A_1R_1+A_1R_2=A_1A_2=b=$the sum of the roots of $x^2-bx+c=0$, and that $A_1R_1\cdot A_1R_2=A_2R_1\cdot A_2R_2=A_2B\cdot A_2A_3=1\cdot c=$the product of the roots.

The famous Greek problem of the duplication of the cube, the finding of the edge of a cube whose volume is twice that of a given cube, amounts to finding a solution of $x^3-2=0$. A solution, doubtfully attributed to Plato (*c.* 400 B.C.), but of

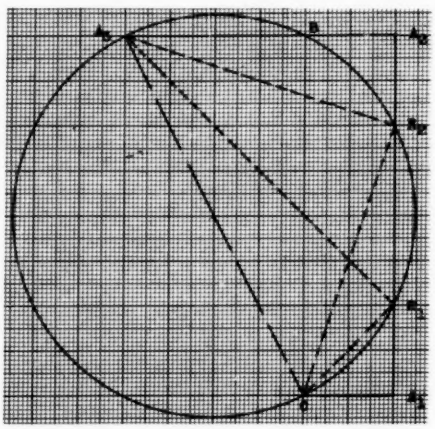

FIG. 3

early Greek origin used two right angle rulers as shown in Figure 4. AO is taken as 1 and $OB=2$. The rulers were then to be slid around until vertices M and N were on AO and OB extended, and the

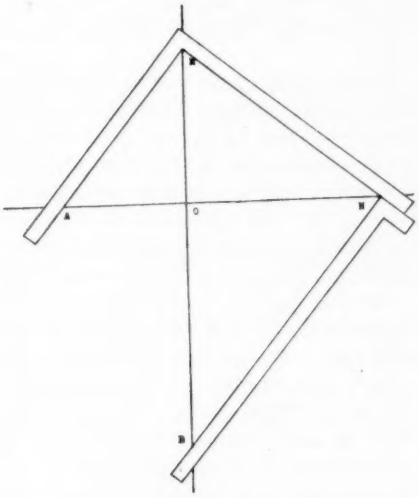

FIG. 4

straight sides passed through A and B as shown. OM is then $x=\sqrt[3]{2}$. The proof follows from the similar triangles which lead to the "continued mean proportion,"

$$\frac{AO}{MO}=\frac{MO}{ON}=\frac{ON}{OB},$$

or, if $MO=x$ and

$$ON=y,\ \frac{1}{x}=\frac{x}{y}=\frac{y}{2}.$$

These proportions lead to the equations of the parabolas $y=x^2$ and $y^2=2x$ which if solved simultaneously give $x=\sqrt[3]{2}$ for one common root.

It is interesting to note that the reduction of the duplication problem to that of finding two quantities in continued mean proportion between 1 and 2 is attributed to Hippocrates of Chios (460 B.C.), and that Menaechmus (350 B.C.) solved it using two parabolas such as those whose equations we derived above. In fact the duplication and trisection problems both stimulated the study of the conics in ancient Greece. Neither parabolas, hyperbolas, nor ellipses can in general be constructed with the compass and unmarked straight edge to which Plato limited pure geometry. The fact that Plato was responsible for this limitation is one reason for doubting that he was the originator of the above solution using two right angle rulers.

Fourth Delegate Assembly

(Continued from page 33)

9. One of the major objectives of the National Council is to extend its services to all areas of the country and to reach individual mathematics teachers everywhere—to see the work of the Council spread among more and more people. How can the Affiliated Groups help the Council in maximally approaching this goal?
10. Is there apparent interest among high school students in a mathematics magazine or other mathematics publication written on their level of understanding and with their interests in mind?
11. Junior memberships in the National Council are now available to students of mathematics in our State Teachers Colleges and other Schools of Education. What interest is being shown in these memberships by the students?
12. Are there other special services which National Council should render directly to under-graduate students?
13. In what specific ways can the Affiliated Groups assist in bringing about improvement in the professional standards and training of mathematics teachers?

APPLICATIONS

Edited by SHELDON S. MYERS
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Ar. 24 Gr. 5–8 *Jimmy Goes Big Game Hunting, a Short Story in Zoorithmetic**

Jimmy, a sturdy boy 4 feet and 6 inches high, was the 11-year old son of an African wild animal trapper. After 3 years of begging and pleading, he finally got his father's permission to go along on one of his hunting expeditions into the jungles of the Belgian Congo.

On the third day out of the port of New York on a tramp steamer bound for Benguella in Angola, a Portuguese dependency on the Atlantic coast of Africa, Jimmy was thrilled to see a huge blue whale, the largest and oldest mammal known on earth. After sighting the animal with instruments, the ship's officers told Jimmy that the animal was about 100 feet long and weighed about 80 tons. Jimmy wondered what fraction the whale was of a football field and how many pounds it weighed. Can you tell him? Whales of this size have been known to dive to a depth of 3,200 feet. What part of a mile is this? When Jimmy's steamer neared port in tropical waters, he caught sight of a 30 foot killer whale. What was the ratio of the length of the killer whale to that of the blue whale? Jimmy kept a notebook of facts about every animal he saw on the expedition. Can you tell him two ways to write the ratio of lengths of the two whales? (With a fraction and with a decimal.)

After landing at Benguella, Jimmy and his father formed their safari and headed northeastward into the teeming jungles of the Belgian Congo.

At first Jimmy was awed by the chattering of monkeys, the screeching of brightly colored tropical birds, and other strange jungle sounds, but soon he got used to this weird world of plants and animals. Some of the vines dangled $13\frac{1}{4}$ feet from $18\frac{1}{2}$ foot branches. How far below the top of Jimmy's head would these vines extend?

One day Jimmy was creeping stealthily toward a small jungle river when he caught sight of a small hippopotamus eating grass in a grove of trees near the bank of the river. While Jimmy was estimating that the mean diameter of this hippo was about 2 feet, he suddenly noticed a huge python slithering out along a limb overhanging the hippo. The snake began to lower its head down toward the hippo, and Jimmy was about to cry out, when his father crept up beside him, took quick aim with his rifle and fired into the head of the reptile. The snake fell to the ground, while the hippo lumbered down the bank and splashed noisily into the jungle stream. Jimmy and his father measured the snake and found it to be 44 feet in length. Jimmy wrote the estimated size of the hippo in his notebook along with the length of the snake and then figured how many times the snake would have been able to wrap itself around the hippo. How many times did Jimmy find?

Jimmy's father told him that the average weight of the hippopotamus is around 8,000 pounds, while that of the elephant, which they would soon meet, is around 9,860 pounds for the male animal. Jimmy computed in his notebook how much more the average weight of elephants is over the hippo's. What did he find? Jimmy's father also told him that, in zoos, the elephant eats about 150 pounds of hay, bran, and

* The department editor would like to express his appreciation to Donna Gibson of the Columbus Zoo who supplied the numerical data about animals that went into this story.

grain per day; the hippo on the other hand, eats 100 pounds of corn, hay, and roots per day. Which animal out-eats the other and by what ratio? Are these animals herbivorous or carnivorous?

Soon Jimmy's safari entered elephant country where they began approaching a herd of medium size. Jimmy's father planned to shoot a large male elephant for its ivory tusks. Together they crept toward the herd from the lee side, so that the wind would blow their scent away from the sensitive trunks of the elephants. Soon they spotted a large bull elephant 12 feet high and began stalking it. The huge beast was about 35 years old with ears three feet wide, which were gently waving back and forth as the animal fed on the grasses between the trees. When they had gotten about 100 feet from it (How many yards was this?), the wind shifted and the animal suddenly raised its trunk and pointed it in their direction. Almost immediately the animal gave a warning trumpet, and the herd began to stampede away in panic. Jimmy's father took quick aim with his elephant gun and fired, whereupon the wounded beast, trumpeting wildly, whirled around and charged down upon them. Hurriedly Jimmy's father took careful aim and fired from the other barrel, bringing the elephant down just 15 yards from their position. (How many feet was this?)

Their native bearers were greatly excited and crowded around the fallen elephant. Using a tape measure Jimmy found that the right tusk was $9'4\frac{1}{2}''$ long and the left tusk $9'$ long. How many inches long were both tusks together? The combined weight of both tusks was later found to be 240 pounds. What was the average weight of each tusk? Since the tusks were curved, how did Jimmy measure their length? While the natives were busy skinning the carcass and cutting it up for food, Jimmy's father told him that elephants in zoos drink about 40–50 gallons of water a day. Jimmy entered this figure in his notebook and also put down how many quarts this would be. Can you tell him? Jimmy remarked about some of the baby elephants he saw in the herd, and Jimmy's father replied that a baby elephant at birth weighs from 150 to 200 pounds. A 160 pound baby elephant would be what per cent of an 8 pound human baby?

The safari moved on and soon came to more open country, known as "veldt" in Africa. One day Jimmy excitedly called his father's attention to 3 giraffes grazing on some trees about $\frac{3}{4}$ of a mile away. How many yards was this? Together Jimmy and his father made their way very carefully toward the extremely sensitive creatures. One of them turned out to be a magnificant creature which they estimated was $18'$ from the crown of the head to the feet. Jimmy was surprised to learn that a giraffe has as many bones in his 7–8 foot neck as any other mammal. Jimmy wondered what startled the animals, for when he and his father were still 60 yards away, they suddenly alerted and ran off with long strides across the grassy plain.

While the safari made its way along the open plain with tall, tan grasses and thorny thickets, Jimmy's father warned him not to stray far from the safari because they were in dangerous lion country. Sure enough, in about an hour Jimmy caught sight of three of the tawny beasts as they bounded away from an antelope carcass and disappeared into the tall grass. Since the lion population had multiplied so rapidly in this region, the big cats were making heavy inroads on the native game supply. Consequently the government at Leopoldville had placed a bounty on each lion pelt brought in.

Jimmy's father decided to try and get at least one of the three lions which they had seen. With Jimmy and three gun bearers, he set out to stalk the elusive beasts. One of the gun bearers was an experienced lion hunter and warned them when they were nearing the animals. After about an hour, they heard the peculiar hoarse cough of a male lion in a nearby

tangle of grass. Immediately one of the bearers uttered a cry of warning as a ten foot lion, including a three foot tail, burst forth from the grass and charged toward them with long graceful leaps. Jimmy's father coolly waited until the beast was about 25 yards away and then dropped him with one shot through the eyes. The lion was a large specimen weighing 500 pounds. What fraction of a ton did Jimmy enter into his notebook? The animal measured three feet from the shoulder to the feet. How many yards was this? Jimmy's father estimated that the lion was a mature animal from 4 to 5 years old. By how many pounds did this lion exceed the average female lion of 300 pounds?

A short time after the lion adventure, the safari was fording a shallow river when Jimmy had a narrow escape from the largest reptile, the crocodile. His father had already crossed to the far bank and Jimmy was waist deep in the center of the stream, when a native porter suddenly shrieked a warning and pointed upstream. Just twenty feet away a huge crocodile was swimming directly towards Jimmy, who froze in his tracks. With the long experience of a seasoned hunter, Jimmy's father took quick aim from the bank 40 feet away and killed the animal with a shot through the head. Immediately a streak of red preceded it downstream, and the crocodile turned over on its back. Several native porters dragged the monster to the shore where Jimmy counted 29 sharp teeth in its long narrow jaws. How many dozen and fraction of a dozen was this?

One of the last events of Jimmy's big game hunting expedition was the capturing alive of a pigmy elephant standing just $2\frac{1}{2}$ feet high and weighing 400 pounds. How many times heavier is the average male elephant than this? The pigmy elephant, which Jimmy named "Gimpy," was transported to a river port on a heavy duty, two-wheeled cart borrowed from a Belgian agent in a nearby native village. While two domesticated buffalo pulled the cart along, how much of Gimpy's weight did each wheel have to support, if he stood midway between them? At the river port, Jimmy's father sold Gimpy to men who bought animals for American zoos.

This marked the end of Jimmy's big game hunting expedition, but he was not unhappy, because he looked forward to showing his notebook full of facts and figures about animals to his schoolmates in the elementary school back home. Which animal that Jimmy encountered was the most interesting to you? Try to find more figures about it in your school library.

Ar 25 Gr. 6–9 *Election Returns*

It seemed that the televised election returns after the November 4 presidential election gave excellent opportunity for practice in the following arithmetic processes:

1. The mental estimation of the approximate ratio of two large numbers representing returns.
2. The mental estimation of the approximate difference between two large numbers.
3. The reading of large numbers.
4. The significance of place value in judging the importance of an estimated difference between two returns.
5. Rounding off large numbers while comparing them.
6. The principles of random sampling when judging the significance of early city returns as compared with later rural returns.
7. The growing importance and versatility of electronic computers. For older students—a number system based on "two."

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
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The A-Shaped Level

AN ISOSCELES triangle is one of the simplest symmetric figures. One of its properties is that the line through the intersection of the equal sides and through the midpoint of the base is perpendicular to the base. Remembering that a level line makes right angles with a plumb line, we can see that a simple way to test whether something is level is to rest an isosceles triangle on it. Then if a plumb line through the vertex hangs directly across the midpoint of the base, we may conclude that the base line is level.

If the sides of the triangle are extended equal distances beyond the base, the figure assumes the shape of a letter A, and the instrument is easier to handle. Levels of this type were used in Egypt. In fact, at least two of them are exhibited in the Museum at Cairo. A-shaped levels are carved on the tombs of Roman engineers in the once-Roman cities of Arles and Nîmes in the south of France. One may suppose that such instruments were used in constructing the pyramids in Egypt and the aqueducts of Rome. This type of instrument did not belong exclusively to the ancient world for just such a level was used in running the Mason-Dixon Line between Maryland and Pennsylvania in 1767.

Construct an A-shaped level. Try it out. You will find that it is reasonably sensitive. Show it to a junior high school class. Show it to a class in demonstrative geometry. Challenge the students to discover why it works as it does. They will enjoy it. So will you.

Two Postscripts on the Calendar. Apropos of the article "September hath XIX Days" which appeared in THE MATHEMATICS TEACHER for May, 1952, DR. H. T. R. AUDE, professor emeritus at Colgate University states that in Denmark, taxes are due on the eleventh of the month rather than on the first or fifteenth. Since Denmark shifted to the Gregorian Calendar in 1700 when the difference between Old Style and New Style was ten days, it seems logical to suppose that the tax date was set ten days later in the calendar just as, a half century later when the discrepancy was eleven days, the tax date in England was changed by eleven days.

Professor A. DAY BRADLEY of Hunter College says:

> Recently I noticed the following tombstone inscription in the cemetery at the Reformed Dutch Church at New Hackensack, Dutchess County, N. Y.
>
> In Memory of William Hoogland
> born 17th December 1701 O.S.
> and departed this life 27th August 1785
> aged 83 years, 8 months and 10 days.
>
> It would seem that not all the difficulties of old and new style were conquered in 1752.
>
> The numbering of the months by the Society of Friends is another curious matter. Before 1752, the Friends or Quakers designated January as 11th mo., February 12th mo., and all of March as 1st mo. Hence the last day of twelfth month was followed by the first day of first month of the same year.
>
> 12th mo., 28, 1701/02, followed by 1st mo., 1, 1701–02. The 24th day of first month was followed by the 28th of first month of the following year.
>
> 1st mo., 24, 1701/02 followed by 1st mo., 25, 1702. This causes a great deal of confusion when old Quaker records are transcribed, especially for dates before September 1752 but recorded after 1752.

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by

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and

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EXPLORATORY DEVELOPMENT OF MATHEMATIC KITS

In response to numerous requests for concrete materials by teachers of secondary mathematics in the San Francisco public schools, two mathematics kits have been constructed for exploratory use. One of these kits has been in use for several months and many teachers are enthusiastic about its possibilities. Kit No. 2 was made available to each school at the beginning of the 1952 fall term.

Ideas for the design of the kits were gathered from teachers who have used some of these devices and from members of the in-service class in secondary mathematics curriculum. Plans were drawn up and submitted to the Samuel Gompers Trade School and James Lick Junior High School for construction.

Kit No. 1 consists of a wooden box containing various plane figures made from one-eighth inch masonite. Other items to be added and used with the kit include rulers, tape measure, scissors, paper, and other objects which will aid in its use. The following figures are included in Kit No. 1:

25 one-inch squares
1 twelve-inch square
2 isosceles right triangles
1 rectangle
2 right triangles (rectangle cut diagonally)
1 obtuse triangle
1 isosceles triangle
1 equilateral triangle
1 parallelogram
2 trapezoids (same size)
1 pentagon (regular)
1 hexagon (regular)
6 circles (various sizes)
16 segments of a circle

Some of the concepts which will be made more meaningful with the aid of the kit are the following:

1. Meaning of: shape, length, width, altitude or height, angle, parallel, perpendicular, perimeter, circumference, area, obtuse, acute, right triangle.
2. Geometry in real life.
3. Approximate measurement.
4. Meaning of pi.
5. Formulas for perimeters, circumference, and areas.

Most effective learning is very often an outcome of the activities of pupils with objects they can see and handle. Following are some of the suggested activities with Kit. No. 1:

1. Handle, compare, identify, and label each object.
2. Cut out similar object from tag board or other material.
3. Collect pictures of geometry in real life and compare with objects in the kit.
4. Practice with ruler, measuring edges, altitudes, perimeters, circumferences.
5. Use one-inch squares to develop meaning of area.
6. Make additional kits from the model.

Kit No. 2 will be used as an aid in teaching concepts of volume and solid measure. The kit consists of a cube made from three-eighth inch plywood, 12 inches on each inside edge so that the volume is one cubic foot. The top, which serves as a lid, is grooved at one-inch intervals giving a checkerboard effect of 144 square inches. The cube has additional use as a storage box for the following items:

144 one-inch wooden cubes
1 cylinder of light tin with a diameter of $4\frac{1}{4}$ inches and height of $6\frac{3}{4}$ inches
1 cone of light tin having the same dimensions as the cylinder
1 quadrangular prism of light tin with a square base 4 inches by 4 inches and a height of 6 inches
1 quadrangular pyramid of light tin with the same dimensions as the prism

The templates for the above metal models were drawn out on stiff paper and traced on tin in the sheet metal shop. They were then cut out, formed, and soldered in order to make them water-tight. A coat of orange lacquer was then applied with a spray gun.

Other materials and instruments which will be added to the kit include a jar of sand, ruler, tape measure, and additional common solids that pupils may make and collect.

Properties of solids, volume, capacity, cubic inch, and cubic foot will be made more meaningful through the use of the kit. Various solids may be formed using the one-inch cubes and the volume formulas developed from them. A laboratory demonstration using sand to fill the cylinder with the cone as a unit of measure is most effective in developing the one-third ratio between the volumes. A similar demonstration may be used with the prism and the pyramid.

Another device which will assist in making the mathematics classroom more of a laboratory is the model transit. Field work in mathematics can be taught in all grades in the junior and senior high schools. With a transit, indirect measurement and scale drawings in the junior high school and solutions to the same problems by trigonometry in advanced classes become real experiences and not abstract theory. Such a transit was used experimentally in several classes in two San Francisco secondary schools in the spring of 1952. Both student and teacher interest was such that arrangements were made to have home-made transits constructed for each of the junior and senior high schools in the city. These transits will be ready for use at the beginning of the 1952 fall term.

A third mathematics kit is in the final stages of preparation. This kit will consist of over 150 recreations in mathematics and will be coded to indicate how they are to be used. These trick problems and puzzles will be printed on 3″ by 5″ cards and will be placed in a file box of the teacher's desk. There are three types of problems in the kit: 1) General Arithmetic, which may be solved by reasoning and which the student tries to solve at his desk; 2) General Teacher puzzle, which will be read to the class by the teacher; and 3) Algebra problems, which will be taken to the desk for solution. The teacher will have a complete set with answers. The problems for the kit were collected by the curriculum seminar, edited and classified by a small committee and submitted to the curriculum assistant for printing. Over 100 of these kits were assembled for distribution to mathematics teachers at the beginning of the 1952 fall term.

SIGURD B. WENAAS
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A MATHEMATICS CLUB FOR THE ABLE

In a recent article in THE MATHEMATICS TEACHER entitled "A Mathematics Program for the Able" by G. Baley Price,* it was emphasized that there is an acute shortage of mathematicians and scientists. If it is assumed that (1) one of the principal reasons for this shortage is that the capable student is not inspired toward science at an early age and (2) the high school is the place to begin such inspiration, then the following type of mathematics club program may be a significant factor in alleviating this shortage.

At the beginning of the 1951 fall semester I had a solid geometry class of seven exceptional students. As is customary, the first few weeks were spent in reviewing plane geometry. Numerous questions were asked concerning the logical nature of geometry. It was decided that we would organize a club in which we could discuss these questions at greater length.

With the aid of John W. Young's *Fundamental Concepts of Algebra and Geometry*, A. Taraski's *Introduction to Logic*, and W. S. Jevons' *Elementary Logic*

* G. Baley Price, "A Mathematics Program for the Able." THE MATHEMATICS TEACHER, XLIV (October 1951), 369–76.

the first three months were spent in discussing the logical nature of mathematics. First, the Aristotelian syllogism was investigated. Next, the various methods of proof, with emphasis placed on the indirect proof, were studied. Theorems from plane geometry were used as examples. After the logical necessity and nature of undefined and defined terms, and axioms and theorems were understood, it was decided that each member would choose a project. These projects were written in the form of a research paper. It was further decided to have a formal meeting, with guests, at which each paper would be read and discussed. Also, after five such meetings, the papers are to be bound and placed in the library with each contributor's name and list of articles in the card catalog file. At the present time, there has been one meeting.

The papers presented at this meeting were "Concerning the Moebius Band," "An Exception to Euclid's Fifth Common Notion," "Concerning Determinants," "An Introduction to the Theory of Groups," "Number Groups," "Hilbert's Approach to Congruence," and "The Fundamental Operations Defined by Geometrical Construction."

The paper entitled "An Exception to Euclid's Fifth Common Notion" concerns a one-to-one correspondence of the points on one leg of a triangle to the points of a segment whose length is the sum of the three legs of the same triangle by using the old concept of "compact" (between any two points there exists another point) thus showing that it is not always true, depending on the definitions of the quantities, "that the whole of a quantity is equal to the sum of its parts and is greater than any of its parts." This idea was actually discovered by a junior student while the club was discussing the characteristics of the line segment.

The second paper is a straightforward demonstration that determinants may be used in solving simultaneous linear equations in two unknowns and of the second order. This problem was assigned to a student who solved it without the aid of reference material.

"An Introduction to the Theory of Groups" was written by a sophomore student who was fascinated by the drawings in a German text concerning group theory. The introduction to the paper discusses the relationship between the elements and operations of algebra to the elements, operations, and four axioms of groups. To demonstrate the axioms, he used a triangle for his element and the operation was a set of three rotations and three reflections. Although the theory of groups is rather abstract and is a course which is usually offered in the senior year at college, this student has a clear picture of its elementary concept. Galois was only nineteen or twenty when he made his contributions to this subject.

The paper entitled "Number Groups" investigated various sets of numbers to see if they formed groups. Some of the sets included were the rational numbers under multiplication and addition, the integers under multiplication and addition, the negative integers under addition, etc. This student and the one who wrote the previous paper are now working on their second papers entitled "Isomorphic Groups" and "Sub-groups and Substitution Groups" respectively.

The next paper was the result of a class discussion concerning other types of geometrics. The question was asked if the discoverers of these other geometries used the same method that Euclid had used in proving two triangles congruent. The teacher gave the class three axioms and a definition and proved the "side angle side" theorem. One of the students then proved the other two theorems, which resulted in his paper entitled "Hilbert's Approach to Congruence."

The last paper, entitled "The Fundamental Operations Defined by Geometrical Construction," was, unlike the others, based on research done in the library.

Besides the two papers previously men-

tioned concerning group theory, some of the forthcoming papers include such subjects as topology, theory of knots, point set theory, countable and uncountable sets. Although these subjects are material for graduate courses, only the elementary aspects of these subjects are studied and the degree of rigor is small.

Perhaps the reason why there is so little interest shown in mathematics in the high school and so few students majoring in mathematics in college is that mathematics is largely a process-developing and problem-solving course; therefore, the student is led to believe that mathematics is only a "helping hand" to the sciences. The writer is aware that modern geometry, topology set theory, group theory, and function theory can not be presented in formal courses in the high school or even in the undergraduate level: but if we are to acquaint the student with the true nature of mathematics, we must tell them in some way what these subjects are because they comprise the fundamental branches of mathematics. Since we do not do this, we are losing capable students to other fields. This idea is comparable to selling a product but not informing the prospective purchaser what he is buying.

In conclusion, one objective of the club is to inspire and develop the capable student toward creative research in mathematics. If enthusiasm and time devoted to their projects are criteria, the club is attaining this objective. Another objective is to acquaint the student with that *branch* of *science* called mathematics with the view in mind of possibly motivating that student to pursue subsequently this branch of knowledge on a professional level, thus, perhaps alleviating our shortage of mathematicians.

JAMES R. BOYD
San Marcos High School
San Marcos, Texas

A GENERAL MOTORS RESEARCH STORY

Milwaukee, Wisconsin, November 6, 1952—Mathematical machines solve problems that engineers and scientists formerly abandoned because life is too short, Arvid E. Roach of General Motors Research Laboratories told the Wisconsin Mathematics Council here today. "In a sense," he said, "modern mathematical devices such as analogue and digital computers have opened an entirely new dimension in scientific and engineering research."

Mr. Roach related how he and a colleague undertook to solve a problem mathematically that had been under experimental study in GM Research Laboratories for more than 15 years. With desk type calculators they worked over two months before they were able to compute enough data to discover whether they were even on the right track. At this point the problem might have been dropped, inasmuch as a complete solution would have taken three years to solve with desk calculators operating eight hours a day and five days a week. "We got in touch with some of our colleagues who are working on high speed computers," Mr. Roach explained. "They agreed the problem was messy but that once it was set up and programmed it could be run off in a few days."

"A computer is incapable of solving any problem which cannot already be solved by hand methods," he stated. "It does not by any means replace the live mathematician who must decide how the problem is to be solved and who must prepare the proper instructions for the machine to follow. Its virtues are simply three. First, it is quick. Second, it does not tire. Third, it does not make mistakes. These are important virtues, but they fall considerably short of the 'Sunday Supplement' machines which are alleged to think in a creative sense."

Not all problems lend themselves to mathematics in industrial research. Rather, some yield to experimental methods, depending on the economics of the situation. In designing automotive engine components, it is often less expensive to make the component and put it through tests until it breaks down. "The cycle of redesigning and retesting may be repeated many times before we are satisfied that we have a design which provides the maximum possible strength with the minimum expenditure of material," he said. "Mathematics would achieve the same results but in some problems it would be vastly more difficult and time consuming than an experimental program. Indeed, it is surprising how often designs which were based only on the designer's intuition and esthetic sense are found by analysis to be mathematically sound. The archery bow is a classic example of the . . . development of a mathematically perfect form."

AIDS TO TEACHING

Edited by

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Boston University
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and

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BOOKLETS

B. 117 Arithmetic in Action

Institute of Life Insurance, 488 Madison Avenue, New York 22, N. Y.

Booklet; 10″×14″; 8 pages; Free in reasonable quantities.

Description: This is a story of an arithmetic class discussing insurance reprinted from *The Instructor* for September, 1951, and filled with many illustrations and tables.

Appraisal: This is a splendid and inspiring article for teachers who wish to introduce material on insurance into elementary school work. It should not be labeled on the cover as "an arithmetic *unit*" for it cannot be so classified under any of the many definitions of that word. It is wordy and loosely organized, but to tighten it up would destroy the narrative quality and personality of the style. A great deal of the material could be put into the hands of the pupils for them to read and interpret. It is especially valuable in the splendid activities which it suggests for the class to do.

B. 118—Mathematical Puzzles

Educational Research Bureau, 1217 13th Street, N.W., Washington 5, D.C.

Booklet; 5″×7½″; 30 pages; $.15 each, discounts for 7 copies and over.

Description: There are 57 numbered items; most of them consist of arithmetic and algebra problems exactly similar to those in most books for the 7th, 8th and 9th grades at the present time. However, item 54 is a table of squares, square roots, cubes and cube roots; item 55 is a method of figuring profit; item 56 consists of area and volume formulas; and item 57 is a confusing account of falling bodies.

Appraisal: This collection is next to worthless. It might have been copied from a textbook of 50 years ago by someone who did not know what it was all about. The arrangement is helter-skelter; the explanations are short and confused. The problems are all of the artificial type. One would do better to pick up an overstock book at some drugstore counter.

CHARTS

C. 41—How High School Mathematics Can Contribute to Your Career

P. S. Jones, Mathematics Department, University of Michigan, Ann Arbor, Mich.

Chart; 8½″×11″; $.05 each, 10 for $.25 (postpaid). Please send money with order.

Description and Appraisal: Two characters are standing before a large signboard which tells them the careers which depend upon college mathematics, which in turn, depend upon high school mathematics. It was originally the frontispiece for a report prepared by a group in Michigan. This is a graphic, interesting, accurate, and thought-provoking chart which is simple enough so that it can be studied through in a few minutes and thus does not scare you away. It also has enough in it to stand explanation and study. Add it to the grow-

ing file you have for your bulletin board.

C. 42—One Square Yard

John C. Winston Co., Philadelphia 7, Pa.

Chart; 37″×37″; Free.

Description and Appraisal: One side of this chart is a grade placement chart of Winston's *Arithmetics*. The other is a square yard divided into nine square feet. Although it says that one square foot is 144 square inches, this is not illustrated and could have been easily done by dividing one of the square feet into square inches.

EQUIPMENT

E. 119—IMOUT (Braille Edition)

American Printing House for the Blind, 1839 Frankfort Avenue, Louisville 6, Ky.

Game; $5.75 plus insurance and shipping charges.

Description: This game is designed to provide drill in the recognition of equivalent fractions (e.g., $\frac{15}{20}$ equals $\frac{3}{4}$, $\frac{6}{18}$ equals $\frac{1}{3}$, etc.). There are 20 large cards with columns of fractions labeled I-M-O-U-T, and small cards with a letter and a fraction on them, and also pegs to put in the holes of the large cards if your fractions are called. As in Beano, one wins by having five pegs in a row.

Appraisal: (The following evaluation was written by Mr. Edward J. Waterhouse, Director of the Perkins Institution and Massachusetts School for the Blind, Watertown, Mass.) The game seems to be very popular with sixth grade pupils. There were some mechanical difficulties, and it was suggested that the pegs and holes should be a little larger and the holes bordered by a raised ring to make them easier to locate. It is also suggested that a set with simpler fractions would be very valuable.

The seventh and eight grades also used the game and apparently did not enjoy it as much. Possibly this was because the sixth grade was handling the reduction of fractions for the first time this year, and the matter was of more current interest.

The difficulty with any game in which blind pupils are involved, is that the game so often is a measure of the finger dexterity of the child. A totally blind child finds it tedious to do a job which a child with a small amount of vision can accomplish rapidly and without effort. This is inherent in almost any activity in which the totally blind compete with the partially seeing, and any improvements which simplify the handling of the pegs would be worth considering.

E. 120—Hit

The Plaway Games, 18 Division Street, Sidney, N. Y.

Arithmetic game; $7\frac{1}{2}$″ ×7″×$1\frac{1}{2}$″; $1.75.

Description: The material consists of 80 pieces of $\frac{1}{2}$″ dowling cut in $\frac{1}{2}$″ lengths and colored, and 25 cards (6″×7″) with letters A through I at the top and J through R at the left. Beside each number is a digit, 1 through 9 being in random order. Directions are given to use the game to drill on the 81 products which are distributed in random order through the center of the card. All cards are alike.

Appraisal: All games of this type are worth knowing about and using to some extent. For the busy teacher, printed cards and colored markers are a great help; for all teachers, sets of these games should be in building or school system collections to illustrate the types of games which are available.

FILMS

F. 72—Borrowing in Subtraction

Teaching Film Custodians, Inc., 25 West 43d Street, New York 36, N. Y.

B&W; 1 reel; 600 feet, 17 minutes. Available only on lease basis.

Description: A boy and girl are selling chickens and try to find out how many will be left if they have 134 and sell 78. In order to figure this out they "think hard"

and then the film turns into a fantasy with their blackboard, chalk and paper plates blown up to a tremendous size. They illustrate 134 with 4 individual plates, 3 piles of 10 plates, and 1 stack of 100 plates and then proceed to subtract, borrowing and regrouping the plates as needed.

Appraisal: The most obvious observation is that this picture carries more of the "Hollywood technique" than most teaching films: a plausible story involving an arithmetic situation, dramatic lighting, imagination to the point of fantasy, lists of screen credits, and touches of humor (usually due to the fantasy). By far the larger number of these are on the credit side, and fewer on the debit. Even though the fantasy is labored, it is enjoyable and to the point. The plot from which the problem grows is convincing and non-obtrusive. One disadvantage of the treatment is the detail which makes a simple problem in subtraction look difficult. Also, there are teachers who will find individual points in the explanation with which they disagree because it differs from theirs. However if more producers would approach their problems with the imagination of this treatment (which does not mean that every film should enlarge the size of the equipment!), there would be more lift and vitality added to our classrooms.

F. 73—Pirro and the Scale

Official Films, Inc., 776 Grand Ave., Ridgefield, N. J.

B&W ($19.75); 1 reel; 8 min.

Description: Throughout this film a man is talking to a marionette which he manipulates in full view of the audience. The marionette is a clown, Pirro, who wanders about a set of scales and a small see-saw which is used for weighing. He weighs different boxes on the scales and hears some discussion of the fact that weight is not the same as size: different sizes may be the same weight. The idea of equal weight is associated with the balancing of a see-saw. Finally a certain amount of weight is indicated on the scales by adding bolts to a box, a few at a time, until the dial reads the desired amount.

Appraisal: The clown marionette will certainly attract attention in the elementary grades even though there is some material on shelves in the background which is not used and therefore distracts. The monotony of the same scene and technique of manipulating Pirro may pall especially in the light of the vapid dialogue. The leisurely pace of discussing the ideas is probably the correct speed for the grade level at which this film will be used, but there should be some way to slow down the concepts without dragging the interest. The actual principles of the scales (except that "different sizes may be the same weight") are not clearly stated or illustrated. There seems to be a need for more planning to get the most good out of this length film; the randomness is excusable in a classroom where pupil reaction intervenes, but not in a film. The lack of underlining of the facts to be learned is not fatal to this film's usefulness if the teacher does so in her follow-up, but how many times must a teacher use the film before this follow-up is thus made useful? Even a teacher's guide would help, but this does not entirely relieve the film of the responsibility to be clearer.

PICTURES

P. 12—Mathematics in Nature

Scripta Mathematica, Yeshiva University, Amsterdam Ave. and 186 Street, New York 33, N. Y.

Two plates; $6\frac{1}{2}'' \times 10''$; $.10 each.

Description and Appraisal: Each plate contains 12 hexagonal designs which are enlargements of snow crystals from Bentley and Humphrey's *Snow Crystals* published by McGraw-Hill. At first glance they seem trivial until you become fascinated with the complexity, symmetry and beauty of the designs. They should cer-

tainly inspire geometry students to attempt to construct such designs for themselves and to see how hard it is to excel nature.

P. 13—Portraits of Mathematicians

Scripta Mathematica, Yeshiva University, Amsterdam Ave. and 186 Street, New York 33, N. Y.

Ten postcards; $3\frac{1}{2}'' \times 5\frac{1}{2}''$; Black and white; $.50.

Description and Appraisal: The set includes pictures of Plato and Aristotle, Pythagoras, Descartes, Newton, Pascal, Lagrange, Fermat, Euler, Kepler, and Galileo. They are printed on good, stiff cardboard, very clear and attractive. For most schools these smaller, less expensive sets of pictures of mathematicians will be easier to store and use than the larger sets (P. 5, Nov. 1949) although not so attractive on the bulletin board. It is to be hoped that more and more will be available and that the mathematicians of interest at the secondary school level will not be neglected.

P. 14—Mathematical Designs

Scripta Mathematica, Yeshiva University, Amsterdam Ave. and 186 St., New York 33, N. Y.

Twenty postcards; $3\frac{1}{2}'' \times 5\frac{1}{2}''$; Black and white; $1.00.

Description and Appraisal: These patterns are curves and repeated designs based upon mathematical shapes and equations. They were drawn by Baravalle, Boyd, Gyllstrom, Moritz, Heath, Stancliff, and El-Milick. They are absolutely hypnotic in their regularity, repetition and complexity. Yet the mathematics which organizes them is explained and thus serves to simplify the complex. A very worthy addition to a bulletin board file. It is to be hoped that someone will put out similar sets in color.

REGISTRATIONS* FOR THE TWELFTH SUMMER MEETING OF The National Council of Teachers of Mathematics held jointly with The New England Institute for Teachers of Mathematics at the Philips Exeter Academy, Exeter, New Hampshire, August 21-24, 1952

Alabama	1	Maryland	6	Pennsylvania	12
California	2	Massachusetts	118	Rhode Island	9
Connecticut	15	Michigan	3	Texas	3
District of Columbia	9	Minnesota	2	Vermont	7
Florida	1	Missouri	1	Virginia	8
Georgia	3	New Hampshire	85	Wisconsin	2
Illinois	6	New Jersey	15	Canada	5
Indiana	6	New York	48	Canal Zone	1
Iowa	2	North Carolina	3		
Kansas	1	Ohio	15	Total Registration	400
Maine	10	Oklahoma	1		

* The above report includes only those who registered for the joint part of the meeting. It does not include members of families who registered for rooms only. Nor does it include those who attended only the sessions of the New England Institute remaining after the close of the joint meeting.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

Anyone who has a learning aid which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. Or if that seems too time-consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

A METHOD OF EXHIBITING THE THEOREM OF PAPPUS IN THE CLASSROOM

Suppose A, E, and C (in this order) are three round-headed screws driven almost home in the wooden frame at the top of a blackboard (Fig. 1). The three screws must be collinear and should be located so that AC is about two feet and AE is not equal to EC. (These screws may be allowed to remain as permanent classroom equipment.)

Procure a narrow piece of pine board and drive three screw-eyes into one edge at points D, B, and F (in this order). The eyes should be in syzygy—the way Penelopeia had the axe-heads set up. Locate the screw-eyes so that DF is about three feet and DB is not equal to AE, EC, or BF.

Now take about fifteen or twenty feet of white string and lace it around the round-headed screws and through the screw-eyes in the order A, B, C, D, E, F, and A. Finally tie the two ends together. When the string is drawn tight the intersections of the three pairs of lines, AB and DE, BC and EF, and CD and FA, will be collinear. In this way it is possible to illustrate mechanically the truth of the Theorem of Pappus. The usual statement of this theorem is as follows: If the vertices of a hexagon lie alternately on two straight lines, the points of intersection of its pairs of opposite sides are collinear.[1]

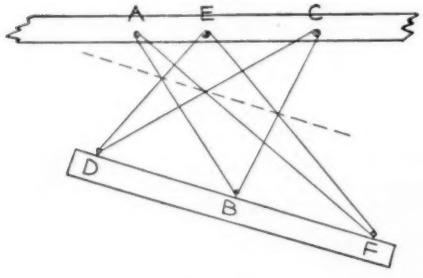

FIG. 1

The student can test the truth of the theorem by holding the board DF (so that the string is tight) with one hand and applying a testing straightedge with the other.

An optional addition to the device would be three small red plastic rings strung on to identify the three crossings which ought to be collinear.

The teacher should observe that the constancy of the sum, $AB+BC+\cdots+FA$, is solely a part of this device, and is *not* a part of the ancient theorem. By drawing some figures the exploring student can find out whether the theorem (or the mechanism) fails when B and F change places.

NORMAN ANNING
University of Michigan
Ann Arbor, Michigan

A SKEW QUADRILATERAL

A handy little device that can be used in a variety of ways in both plane and solid geometry is one that may for the sake of

[1] This statement of the Theorem of Pappus is taken from the textbook, *An Introduction to Modern Geometry*, by Levi S. Shively, New York: John Wiley & Sons, 1939, p. 110.

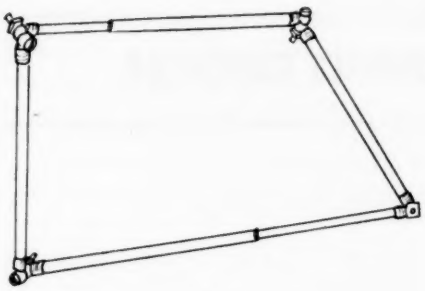

FIG. 2

convenience be referred to as a skew quadrilateral device.

The only materials needed to build the device are four short extensible curtain rods and four $\frac{1}{8}''$ stove bolts $\frac{3}{4}''$ long. The length of each rod when collapsed to its shortest length is 12". The total cost of the four rods at any department store household counter is about 80 cents.

Assemble the device with stove bolts as illustrated in Figure 2. No tools are needed. Simply join the rods to form a quadrilateral.

The unusual thing about this simple device is its flexibility. Every rod is extensible, and every vertex works like a knuckle joint. The four rods can be made to lie side by side or they may be twisted to form any kind of skew quadrilateral imaginable. Try it!

As a plane geometry demonstration model the device can be used to illustrate how the different kinds of plane quadrilateral figures may be developed from the

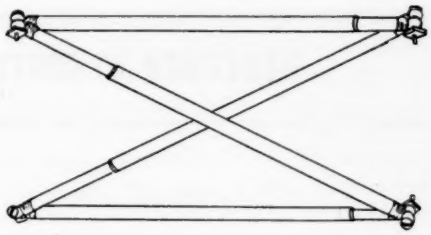

FIG. 3

same general quadrilateral simply by proposing different requirements. Figure 3 shows how the device may be used to form a contra-parallelogram.

As a solid geometry demonstration model the most obvious use of the device is of course that it can be formed into a skew quadrilateral. In addition it may be used to illustrate that a plane is determined by: (1) a straight line and an external point, (2) three non-collinear points, (3) two intersecting straight lines, and (4) two parallel lines. In connection with (2) it may easily be shown with the aid of the device that four points, no three of which are collinear, determine four planes.

By tying elastic between the midpoints of adjacent rods it may be shown that the figure so formed is a parallelogram. The definition of a dihedral angle can be illustrated by connecting any two non-adjacent vertices with elastic. Other uses will suggest themselves to the teacher once he has the device in his possession.

THE MATHEMATICS LABORATORY
Monroe High School

President-Elect DWIGHT D. EISENHOWER: "The United States is facing critical times, and in the coming years we must make the best use of the vigor, enthusiasm, intelligence and technological skill of young people of this country. . . . Creative ability is one of America's greatest assets—one which we cannot afford to waste. Engineers, already in short supply, must make their contribution to the nation's welfare at their highest capacities where their special training can continue to accomplish miracles of production—our principal weapon in defending our society of free men." —from "Youth and Engineering," *The Journal of Engineering Education*, November 1952, pages 131–132.

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
School of Education, New York University, New York 3, N. Y.

The Questions: How did the present elementary mathematics courses for freshmen in arts colleges evolve? What current ideas and forces are influencing the nature of these courses?

The Study: Wahlert, Howard E. *Elementary Mathematics in Arts Colleges.* Ph.D. dissertation. Teachers College, Columbia University. 1952.

The purpose of Dr. Wahlert's study was "to investigate the influences that have affected the teaching of elementary college mathematics; the result of these influences as seen in the expressed aims of college teachers of mathematics; and the extent to which these aims have been achieved in elementary college courses." (page 1)

Although Dr. Wahlert describes elementary college mathematics to be merely "the material covered in freshman courses in American colleges" the curricular scope of the content is far from narrow. In various colleges this material includes "intermediate and advanced algebra, trigonometry, mathematics of investment, statistics, analytic geometry, elementary calculus, and various other topics, . . . , for example, the logical structure of mathematics, non-euclidean geometry, and infinite classes." (page 1)

Since this investigation was primarily historical in type, three periods beginning with the founding of Harvard were selected: 1636–1876, 1876–1914, and 1914 to the present. These corresponded respectively to a tradition-forming era, an interval of stabilization and extension of instruction to higher mathematics, and the modern period characterized by rapid growth in mathematical research and "new disturbing forces" affecting elementary mathematics instruction.

During the colonial period, when relatively little mathematics was known, the content of courses included much of the arithmetic, algebra and geometry of our present pre-college courses. The activities of the early settlers did not draw heavily on the little mathematics that was known. Furthermore, preparation for the ministry, a dominant purpose of the early colleges, did not make severe demands on the "queen of the sciences." Later, however, the spurt in the growth of mathematics and science culminating in Newton's work led to new curricular topics, including fluxions.

During the later colonial years, which reflected less emphasis on religion and more on trade and commerce, the demand for surveying and navigation made more attention to mathematics necessary.

From 1776 to 1825, the first half century of the new nation, civil and secular control of education and the need for more flexibility in educational programs to prepare students for life in a democracy were discussed vigorously. However, the curriculum of the colleges changed little during these debates. Textbooks began to replace manuscripts. At first only English texts and translations of French texts were available. After 1814 American books made their appearance under the authorship of Jeremiah Day, President of Yale, in the fields of algebra, mensuration, plane trigonometry, navigation and surveying.

The expanding needs of the nation in the period from about 1825 to 1875 exerted a pressure for more practical math-

ematics. For instance, Rensselaer began instruction in engineering and technology. The established institutions modified their programs by adding scientific schools or courses. In 1862 the Land Grant Colleges were established by the Morrill Act. These institutions were to include not only the classical studies but were also charged "to teach such branches of learning as are related to agriculture and mechanic arts . . . in order to provide the liberal and practical education of the industrial classes in the several pursuits and professions of life."

An expansion of knowledge accompanied the growth of the industrial, agricultural and commerical aspects of American life. Contact with German universities raised the ideals of academic scholarship. Increased departmentalization of the faculties, the idea of elective courses and a critical examination of traditional education followed. More conservative colleges and faculties defended themselves with the doctrine of mental discipline; mathematics was included to develop reasoning powers. The traditional subjects, it was claimed, serve as a preparation for any sort of vocation or type of living. Entrance requirements began to include arithmetic and by 1875 algebra and geometry. The tendency to move the more advanced mathematics into the earlier years was illustrated in 1852 when Yale transferred analytical geometry and calculus to the sophomore year.

"The contents of textbooks of a hundred years ago are quite similar to recent textbooks of the traditional type." (page 25) About 1850 alegbra was divided into high school and college algebra. Elementary geometry was a college subject until almost the end of the 19th century. Eventually, it became a mixture of Euclid's work and Legendre's *Elements de Geometry*. Davies' *Trigonometry and Mensuration* (1868) contained chapters on logarithms, solution of triangles, analytical trigonometry, spherical trigonometry, and mensuration. Exercises and applications were rare. Olney's *General Geometry and Calculus* (1879) contained four chapters on plane analytic geometry and three on the calculus. The latter dealt with formulas for differentiation, partial differentiation, development of functions in series, indeterminate forms, maxima and minima, and formal methods of finding the anti-derivative.

However, the order of topics, methods of presentation and standards of rigor differed from those of today. In Loomis' *Algebra* negative and zero exponents were introduced in connection with the division of monomials. It was assumed that the laws of exponents automatically apply to the new numbers. The same assumption was made about fractional exponents. Imaginaries were considered "symbols of absurdity." The trigonometric functions were explained as line values rather than ratios. In teaching the calculus some teachers used the infinitesimal method instead of the method of limits.

In general, the educational justification of this content for this fifty year period (1850–1900) was mental discipline with automatic transfer of learning assumed.

During the period, 1876–1910, dominating influences were increasing industrialization; the growth of national wealth accompanied by more endowments to the colleges; the great increase in public high schools from 100 in 1860 to 6000 in 1900; the doubling of college enrollments from 1875 to 1900; the establishment of state universities; the influence of German universities on American scholars; the undermining of the doctrine of mental discipline by new developments in psychology; an increase in the use of electives, and the maturation by 1900 of technical schools and scientific subjects.

From 1875 to 1900 mathematics in the colleges became a subject in its own right, rather than the servant of astronomy and physics. Research was started by the initiation of graduate study, aided and abetted by professors returning from

study at German universities. In 1894 the founding of the American Mathematical Society gave more encouragement to research activity. On the other hand, the training of the mind was still the dominant aim of mathematics instruction as late as 1910. This continued despite the influence of Moore, Felix Klein, J. W. A. Young who called for a more psychological approach in methods and sequences, less manipulative dexterity and greater attention to the importance of mathematics in modern civilization.

The founding of the College Entrance Examination Board in 1900 brought stiffer entrance requirements; criticism of the quality of secondary school instruction was a concomitant. On the other hand, the degree requirements in the colleges decreased with the extension of the elective system; the latter also led to more offerings in mathematics. At the same time, pleas were made for omission of obsolete topics and useless manipulation, and addition of more modern topics and applications.

The recitation method was used until the student reached the calculus. At that time came greater use of the lecture or lecture-and-quiz method. The order of presentation was not a settled issue. One doctrine was the theory that the best sequence would be revealed by the historical development of mathematics; another was influenced by the idea that the different parts of mathematics should be taught according to certain unifying themes. Not much stock was placed in pedagogics; in fact, it was stated that "dogmatic instruction in the art and method of teaching would be injurious and disastrous in collegiate instruction."

The values in learning mathematics described by the Mathematical Association of America in 1923 were the utilitarian, the disciplinarian and the cultural. "The ability to use mathematics in practical situations," "an understanding of the nature of reasoning," knowledge of "the fundamental role of mathematics in past civilizations, and in our present civilization" and appreciation of "the aesthetic character of mathematics" were approved.

Between 1910 and 1949 two competing philosophies for colleges of liberal arts emerged. "The Great Tradition" view classified certain studies, including mathematics, as cultural and ruled out others. The "Experimental Naturalism" doctrine proposed that higher education should involve students in activities requiring the application of scientific method to the affairs of the actual world about them.

In colleges where one year of mathematics is required Dr. Wahlert believes that the plan calling for one year of cultural mathematics reflects best the "Great Tradition" view. Other arrangements are (1) two term courses, (2) an integration of two or three of these courses and (3) a survey course of a preparatory or combination preparatory-cultural types. The cultural course is of the third type. Under (3) there seems to be only a small difference between the preparatory and preparatory-cultural types. It is not clear which of the three arrangments, if any, is consonant with the "Experimental Naturalism" doctrine.

Distribution of Yearbooks. The yearbooks of the Council are no longer handled by the Bureau of Publications of Teachers College, Columbia University, New York City. All orders for yearbooks, including the Twenty-First, should now be sent, with remittance, to the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C.

Student Memberships. Although many student memberships have been received by the Washington office, it is likely that many persons are still unacquainted with this new type of membership. Any student, graduate or undergraduate, who has never taught, may join the Council at the special rate of $1.50 for the first year. Orders must be submitted by a staff member of the college or university who is a member of the National Council.

MATHEMATICAL RECREATIONS

Edited by AARON BAKST
135-12 77th Avenue, Flushing 67, N. Y.

THE TOPIC of systems of numeration, other than the decimal, is usually recommended for inclusion in the courses of study (or curricula, the choice of words is a matter of *pedagogical taste* of the present generation of educationists, although the rose from Romeo and Juliet might be appropriately employed here for elucidation purposes) of secondary mathematics. Generally, there is still very little agreement concerning the proper placement of this topic. Some schools of thought hold that this topic be relegated to mathematical clubs and recreations. This department gladly accepts this topic and it proposes to illustrate one specific phase which, in the opinion of the editor of this department, is not only fascinating but also very instructive. And, incidentally, the development which follows is offered in order to show how simple some mathematical treatments may be, especially when on the surface the solution of a problem seem to be so difficult, so complicated, almost impossible, and if possible, *higher mathematics* is called in as an excuse for "passing the buck."

Some of those teachers who are not yet imbued with the modern ideas of assigning to mathematics the main task of building character, strengthening democracy, developing physical well being, building higher ethical and moral precepts, and all other high sounding aims which relegate mathematics to the junk-heap (if not openly embracing the idea that mathematics should be abolished altogether from the schools), some of these teachers are trying to keep their heads above the deluge of psuedo-pedagogical theories which threaten to engulf our schools to the glorification of self-appointed Pied Pipers of nondescript pedagogical theories. Some of these teachers are still trying to make mathematics work in the schools. This type of mathematical recreation is addressed to those teachers who still feel that they can imbue the pupils with those feelings for the powers of mathematics which cannot be replaced by meaningless (to a high school pupil) discussions of higher economic and social theories. Mathematics may not be the panacea for our present ills, but it is a mighty and powerful instrument in every phase of the modern world. And mathematics, to be appreciated, cannot be learned through the means of discussions and debates.

Let us consider the simple problem of tests for divisibility. Usually, such tests are never taught, except in a "passing by" manner. We may encounter some teacher who will teach the tests for divisibility by 2, 3, 4, and by 9. The test for divisibility by 11 is "so difficult" that "in order to master it one must have an I.Q. of, say, at least 200." The test for the divisibility by 7? Well, in order to master it one must be versed in Higher Algebra, in Number Theory, and for a full measure we might throw in Function Theory, especially the Real Variables. So, the test for divisibility by 7 is relegated to those mysterious domains which are reserved for the higher breeds of mathematicians.

But the problem of the tests for divisibility is really so simple that a rudimentary knowledge of intermediate algebra is sufficient for its mastery. The requirements for this mastery are limited to the understanding of polynomials of the form

$$Ak^n+Bk^{n-1}+Ck^{n-2}+\cdots+Mk^2+Nk+R.$$

This type of a polynomial forms the basis for all systems of numeration.[1] The other property with which one must be familiar is the fact that a^n-b^n is always divisible by $(a-b)$ for all integral positive values of n.

If a and b are different bases for two systems of numeration, then the numbers in these two respective systems may be written as

$$A_1a^n+B_1a^{n-1}+C_1a^{n-2}+D_1a^{n-2}+\cdots+M_1a^3+N_1a+R_1$$

and

$$A_2b^n+B_2b^{n-1}+C_2b^{n-2}+D_2b^{n-3}+\cdots+M_2b^2+N_2b+R_2.$$

The reader will note that the value of n is the same for the two numbers. The reason for this special case will become obvious immediately. Let us assume that two numbers are written in two different system of notation but the same digits are employed in the same sequence, as, for example, $345_{(10)}$ and $345_{(7)}$.[2] We may then write the two polynomials as

$$Aa^n+Ba^{n-1}+Ca^{n-2}+Da^{n-3}+\cdots+Ma^2+Na+R$$

and

$$Ab^n+Bb^{n-1}+Cb^{n-2}+Db^{n-3}+\cdots+Mb^2+Nb+R.$$

Let us obtain the difference of these two polynomials. We have then

$$A(a^n-b^n)+B(a^{n-1}-b^{n-1})+C(a^{n-2}-b^{n-2})+D(a^{n-3}-b^{n-3})+\cdots+M(a^2-b^2)+n(a-b).$$

This difference is divisible by $(a-b)$ because a^n-b^n is divisible by $(a-b)$.

Thus, the problem of the divisibility by $(a-b)$ is determinable for a number written in one system of numeration (a) by the divisiblity of the value of this number (that is, written in terms of the same digits in the same sequence) by $(a-b)$ when this value is computed for the system of numeration at the base b.

Suppose that we wish to determine the divisibility of the number 162 by 7. The number 162 is written in the decimal system of numeration. Let us compute the value of 162 in terms of the base $(10-7)=3$. Note that $10-3=7$. Thus, the second base must be 3. The fact that the digit 6 does not exist in the 3-system of numeration should not confound us. We are interested in a certain result, and we may disregard this violation. We have then

$$1\cdot3^2+6\cdot3+2=9+18+2=29.$$

29 is not divisible by 7, and $162_{(10)}$ is not divisible by 7.

Let us examine the divisiblity by 7 of the number $168_{(10)}$. We have then

$$1\cdot3^2+6\cdot3+8=9+18+8=35.$$

Thus $168_{(10)}$ is divisible by 7.

This test for divisibility holds good for numbers with any number of digits. Let us test the divisibility by 7 of the number 16,305. We have then

$$1\cdot3^4+6\cdot3^3+3\cdot3^2+0\cdot3+5=81+162+27+3=274.$$

We shall now test the divisibility of $274_{(10)}$ by 7. We have then

$$2\cdot3^2+7\cdot3+4=18+21+4=43.$$

43 is not divisible by 7. Thus $16{,}305_{(10)}$ is not divisible by 7. As a matter of fact $16{,}305=2{,}329\cdot7+2$. Note that $43=6\cdot7$

[1] The reader is referred to the book *Mathematics, Its Magic and Mastery*, by Aaron Bakst, D. Van Nostrand Co., Inc., New York, 1952, pp. 9–30 for an exposition of the principles of systems of numeration.

[2] The development of this topic of divisibility is fully discussed in the forthcoming book by the editor of this department *Mathematical Games and Recreations* (D. Van Nostrand Co., Inc.). All rights are reserved.

+2. The remainders in both cases are equal to 2.

Thus, the test for divisibility by 7 may be carried consecutively until a small number (less than 100) is obtained.

The test for the divisibility by 9 is developed in the same manner. The two bases are in this case 10 and 1.

The test for the divisibility by 11 is developed in the same manner. The two bases in this case are 10 and (−1). And why should there be any objections to the introduction of negative bases of numeration?

We may thus develop tests for the divisibility by 13, for example. In this case, one base is 10 and the second base is (−3).

Thus, the tests for divisibility may be successfuly employed for the purposes of evaluation of polynomials. If teachers claim that there is not enough work of numerical nature where the pupils may have some practice in arithmetic, the above examples give ample evidence that such claims are not justified.

There are some other interesting applications of the properties of the principles of systems of numeration. These will be discussed in the next issue of THE MATHEMATICS TEACHER.

NEWS NOTES

An **Institute for Mathematics Teachers** will be held next summer at the **University of Virginia,** Charlottesville, Virginia, **July 27–August 7.** Co-Directors are W. W. Rankin, Emeritus Professor of Mathematics, Duke University and Francis G. Lankford, Jr., Professor of Education, University of Virginia.

The 1953 Science Prize Competition in Mathematics at **Knox College** will be held Saturday, March 7, 1953. Examinations will be held at 10:00 A.M. on the Knox College Campus at Galesburg, Illinois and at 1:00 P.M. at the Chicago Bar Association, 29 S. LaSalle Street, Chicago, Illinois. Two Mabel Heren Prize Scholarships in Mathematics ($1000 and $450) will be offered on the basis of an examination which presupposes a knowledge of algebra and plane geometry but is primarily designed to measure ability and understanding of mathematics. An applicant competing in mathematics may not compete in another field of science, must rank in the highest quarter of his high school class and be recommended by his high school mathematics teacher. A high school may enter three contestants in the field of mathematics and applications, forwarded by the recommending teacher, must reach the Director of Admissions at the College not later than March 3, 1953. C. Robert Wallingford of the Pontiac, Illinois Township High School was the winner of the 1952 contest.

A small number of copies of **Bibliography of Mathematical Works Printed in America Through 1850** by L. C. Karpinski which for several years was listed as out of print were made available recently according to an announcement from the University of Michigan Press at Ann Arbor.

The **Consumer Education Department** of Household Finance Corporation, 919 N. Michigan Avenue, Chicago 11, Illinois, announces the following **booklets:** "Small Loan Laws of the United States" (Free), "Money Management—Your Home Furnishings Dollar" (10¢) and "Consumer Credit Facts for You" (10¢).

BOOK SECTION

Edited by JOSEPH STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

Elementary

Working With Numbers, Book 1, Teacher's Text Edition, by Joyce Benbrook, University of Houston; Cecile Foerster, Houston Public Schools; and James T. Shea, San Antonio Public Schools. Cloth, 88+38 pages, 1952. The Steck Co., Austin 61, Texas. $1.72.

Working With Numbers, Book 1, Teacher's Worktext Ed., by Joyce Benbrook, University of Houston; Cecile Foerster, Houston Public Schools; and James T. Shea, San Antonio Public Schools. Paper, 112+37 pages, 1952. The Steck Co., Austin 61, Texas. $0.64.

Working With Numbers, Book 2, Teacher's Text Ed., by Joyce Benbrook, University of Houston; Cecile Foerster, Houston Public Schools; and James T. Shea, San Antonio Public Schools. Cloth, 111+46 pages, 1952. The Steck Co., Austin 61, Texas. $1.88.

Working With Numbers, Book 2, Teacher's Worktext Ed., by Joyce Benbrook, University of Houston; Cecile Foerster, Houston Public Schools; and James T. Shea, San Antonio Public Schools. Paper, 128+38 pages, 1952. The Steck Co., Austin 61, Texas. $0.64.

Working With Numbers, Book 7, by James T. Shea, San Antonio Public Schools. Paper, 128 pages, 1952. The Steck Co., Austin 61, Texas. $0.56.

High School

Practical Mathematics (Fourth ed.), by Claude I. Palmer and Samuel F. Bibb, Illinois Institute of Technology. Cloth, xii+769 pages, 1952. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $4.50.

College

Basic Skills in Mathematics, by H. Vernon Price and Lloyd A. Knowler, both of The State University of Iowa. Cloth, viii+249 pages, 1952. Ginn and Co., Statler Bldg., Boston 17, Mass. $3.25.

Fundamental Procedures of Financial Mathematics, by Merrill Rassweiler, University of Minnesota; and Irene Rassweiler. Cloth, vii +260 pages, 1952. Macmillan Co., 60 Fifth Ave., New York 11, N. Y. $3.25.

Science and Method, by Henri Poincare. Paper, 288 pages, 1952. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $1.25 (Cloth, $2.50).

Science and Hypothesis, by Henri Poincare. Paper, xxvii+244 pages, 1952. Dover Publications, Inc., 1780 Broadway, New York, 19, N. Y. $1.25 (Cloth, $2.50).

Foundations of Combinatorial Topology, by L. S. Pontryagin. Paper, xii+99 pages, 1952. Graylock Press, P.O. Box 1223, Rochester 3, N. Y. $3.00.

Linear Algebra and Matrix Theory, by Robert R. Stoll, Oberlin College. Cloth, xv+272 pages, 1952. McGraw-Hill Book Co., 330 W 42nd St., New York 36, N. Y. $6.00.

Miscellaneous

Teaming Up for Public Relations, A Handbook for Leaders in American Education. Paper, 48 pages, 1952. National School Public Relations Association, 1201 Sixteenth St., N.W., Washington 6, D. C. $1.00.

Funcoes Ortogonais na Resolucao de Problemas da Teoria da Elasticidade, Vol. 1, by Telemaco van Langendonck, University of Sao Paulo, Brazil. Paper, viii+69 pages, 1952. Associacao Brasileira de Cimento Portland, Sao Paulo, Brazil.

Core Curriculum Development—Problems and Practices, by Grace S. Wright. Paper, vi+104 pages, 1952. Federal Security Agency, Office of Education Bulletin 1952, No. 5. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. $0.30.

BOOK REVIEWS

Growth in Arithmetic, Grade 4, John R. Clark, Charlotte W. Junge and Harold E. Moser. Yonkers-on-Hudson, New York, World Book Company, 1952. v+314 pp., $2.12.

This is an attractive, carefully planned, well arranged text. Large print, well spaced on good paper, makes reading easy.

"Circus Time" is very natural and usable way to start the new term by reviewing time telling and to extend old and new facts into other experiences. Much of the first eighty pages is given to relearning and extension of Third Grade work. In all topics, illustrations and problems are especially planned to encourage independent thinking. The treatment of and emphasis on the interrelationships of the four processes are very good—showing that counting leads to addition and in turn to multiplication and that counting backward leads to subtraction and in turn to division.

The treatment of fractions is very well planned. Children are led to discover generalizations, but no stated rules are given.

Unusually good problem solving sections are given.

One very outstanding feature throughout the book is the happiness expressed on all the faces in the pictures, emphasizing the fact that arithmetic is really fun!—LORENA HOLDER, Dallas Public Schools, Dallas, Texas.

Everyday Mathematics (Rev. Ed.), Harl R. Douglass, Lucien B. Kinney, and Vincent Ruble. New York, Henry Holt and Company, 1951. vii+504 pp., $2.48.

The text is designed primarily for a course in general mathematics at grade nine. However, there is material presented which is also suited to more advanced students. For example, the chapters on security, government, and the home contain material that could be used in a junior or senior course in general mathematics.

There are many admirable features in the text. The following are the most important in the opinion of the reviewer: (1) the text presents an early introduction to problem-solving; (2) problem scales are provided in each chapter; (3) many well chosen applications are used to introduce topics as well as to summarize the topics; (4) ample exercise material is available for fixing important mathematical ideas and operations; and (5) specific emphasis is given to the vocabulary of everyday mathematics. The supplementary material is very challenging and should be valuable in meeting the mathematical needs of the brighter pupil. The text contains some challenging content for the general mathematics course and does a good job of presenting the mathematics of everyday life.—WILLIAM L. CARTER.

Mathematics in Action (Book 2, third ed.), Walter W. Hart and Lora D. Jahn. Boston, D. C. Heath and Company, 1952. vii+294 pp., $2.24.

This book is the second volume in a junior high school mathematics series. It is designed to meet the needs of a wide range of abilities and interests at the eighth grade level.

In leafing through the book I was at once attracted to the clear cut, practical illustrations that introduce carefully selected thought problems and instructive materials. The starred assignments should be especially challenging to the more eager learners.

There is a carefully planned program of diagnostic testing, remedial teaching of the fundamental processes and reviews for the maintenance of these skills. The early stress on the importance of place value and its use in approximating and rounding numbers is especially commendable.

The review and continued teaching of percentage is well done, but even better is the excellent balance between the space given to the applications of percentage and to the useful concepts of plane geometry and mensuration. This is an important factor in retaining pupil interest.

The early introduction of the simple equation in relation to the formula makes it possible to use algebraic meanings correctly throughout the year. A later unit further develops the elementary concepts of algebra.

Although one might take exception to a few minor points in the presentation of new material, in general it is an attractive, well organized piece of work. It is a text book, I believe, that any teacher would enjoy using.—GRACE I. GISH, Western Michigan College of Education, Kalamazoo, Michigan.

New Applied Mathematics (Fourth Ed.), Sidney J. Lasley and Myrtle F. Mudd. New York, Prentice-Hall Inc., 1952. xii+387 pp., $2.56.

This is the fourth edition of a pioneer book in mathematics for non-academically minded high school students. It is designed particularly for ninth-year students, but may be used in the later high school years. The fundamental emphasis is on computation and its applications in business and geometry. Some attention is given to geometrical relationships and constructions, and very elementary algebra. The glossary of mathematical terms and the tremendously large number of practice exercises will particularly appeal to both students and teachers. A comparison of this edition with the original 1932 edition shows the almost complete evolutionary change that has taken place in this area of mathematical education. HOWARD FEHR, Columbia University, New York, New York.

Dictionary of Mathematical Sciences, Vol. 1, German-English, Leo Herland. New York, Frederick Ungar Publishing Company, 1951. 235 pp., $3.25.

Yes, a German-English mathematics dictionary. Although it does not claim to be complete, the author tried to include all the important mathematical terms in algebra, geometry, statistics, commercial arithmetic, physics, etc. In carrying forward this work it was necessary to include many common words often used in a non-mathematical sense but which, nevertheless, had a technical meaning in mathematics. In doing this, however, only the mathematical meanings are given.

The author states: "Wherever necessary, model usages showing the application and grammatical construction of the key word are supplied generously, far beyond the measure commonly allotted them in technical dictionaries. Special efforts have been made to put this work on a par with the standard expected from general bilingual dictionaries. The emphasis everywhere is on clarity and distinction." This appears to be quite true.

Although this is not also an English-German dictionary, it should be quite valuable to one translating from the German to English—especially for such students of mathematics as those studying to pass their German examination while studying for an advanced degree (as the Ph.D.).

For those whose particular interest is mathematical German this is the best German-English dictionary your reviewer has seen.—JOSEPH STIPANOWICH, Western Illinois State College, Macomb, Illinois.

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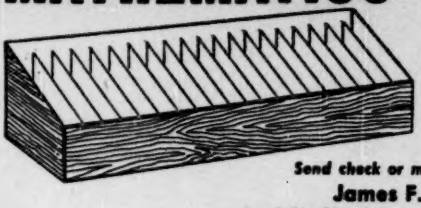

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